Some Classes of Repeated-root Constacyclic Codes for Multi-symbol Read Channels and Applications

Madhu Kant Thakur

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About this Monograph

This monograph is a part of the dissertation entitled "DISTANCE DISTRIBUTIONS OF SOME CLASSES OF REPEATED-ROOT CONSTACYCLIC CODES FOR MULTI-SYMBOL READ CHANNELS AND APPLICATIONS IN MDS CODES" submitted for the award of the degree of Doctor of Philosophy (Ph.D.), to the Indian Institute of Technology (Indian School of Mines), Dhanbad; in September 2022 under the guidance of following supervisors-

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“To my mother and father, who never stopped believing in me.”

Abstract

The main objective of coding theory is to find codes with efficient encoding and decoding techniques and the greatest possible value of distance for a given code length, code size, and code alphabet cardinality. Various distances like Hamming distances, symbol-pair distances, etc., have been established and investigated in coding theory to analyse a code's error-detecting and error-correcting capabilities with respect to various communication channels. A maximum distance separable code (MDS code) is a code that meets the Singleton bound. The MDS codes have the greatest error-correcting capability. When the length and size of the codes are fixed, an MDS code has the largest distance. As a result, nowadays, researching and discovering MDS codes with respect to various distances is a hot topic in coding theory.
In the theory of error-correcting codes, constacyclic codes over finite fields play a crucial role. Constacyclic codes, moreover, have many real-world applications. Shift registers can effectively encode and decode these codes because they have rich algebraic structures. They also offer excellent error-correction capabilities. All of this clarifies their chosen engineering role.

In this monograph, we determine the multi-symbol distances of various classes of repeated-root constacyclic codes over some classes of finite commutative chain rings and investigate various classes of non-trivial MDS codes using these multi-symbol distance distributions. Let $\mathbb{F}_{p^m}$ be the finite field of order p^m, where p is an odd prime, and m is a positive integers. The symbol-pair distances are completely determined for some classes of λ-constacyclic codes of length $4p^s$ over $\mathbb{F}_{p^m}$, where s is a positive integers and $\lambda \in \mathbb{F}_{p^m}^*$. We explore two new classes of non-trivial MDS symbol-pair codes of length $4p^s$ and exhibit several new MDS symbol-pair codes of length $4p^s$

over the finite field $\mathbb{F}_{p^m}$ as examples.

Next, we consider some classes of constacyclic codes over the finite commutative chain ring, $\Re = \mathbb{F}_{p^m} + u\mathbb{F}_{p^m}$ with $u^2 = 0$. For any invertible element Λ of $\Re$, the symbol-pair distances of all Λ-constacyclic codes of length $2p^s$ over $\Re$ are completely obtained. We identify all MDS symbol-pair constacyclic codes of length $2p^s$ over $\Re$. The Hamming and b-symbol distances of all Λ-constacyclic codes of length $4p^s$ over $\Re$ are completely determined for any non-square unit Λ of $\Re$. As examples, several good codes with new parameters are constructed. We also identify all MDS constacyclic codes of length $4p^s$ over $\Re$ with respect to the Hamming distance as well as the b-symbol distance. Also, we construct some non-trivial MDS b-symbol Ω-constacyclic codes of length $4p^s$ codes over $\Re$ with respect to b-symbol distance for $b = 4$, where $\Omega \in \mathbb{F}_{p^m}^*$ are units of $\Re$.

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