

3D Geological Modelling and Management System

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ABSTRACT

A three-dimensional (3-D) geological model has been established for Hong Kong using existing borehole data in order to facilitate detailed site investigations for future engineering projects. This study aims to digitalise ground investigation data in Hong Kong, develop easy-to-use tools for 3-D borehole management and visualisation, and eventually establish 3-D geological models for Hong Kong. The modelling capabilities include geological data retrieval and processing, geological cross-section creation, fence diagrams and 3-D model construction. With approximate 90,000 boreholes processed, 3-D virtual boreholes can be created and managed using ArcGIS Pro. Further, cross-sectional diagrams, fence diagrams and 3-D models can be created and presented. The 3-D geological model established shows the complexity of Hong Kong geological formation layers. Building a 3-D geological model based on machine learning or artificial intelligence is proved to be a feasible way to provide an accurate evaluation of soil layering. The interpreted cross-sections and constructed fence diagrams help engineers and geologists to better understand the complicated sub-surface profiles in a 3-D way, and provide estimates of the volumes of different types of soil locally. The 3-D model will become a design tool for future city and infrastructure planning and constructions.

Keywords: 3D model, Machine learning, ArcGIS

1 Introduction

Hong Kong has a high population density and lacks the supply of flat lands, inducing associated social issues such as housing shortages. To meet the demand, decentralize the population from over-occupied urban areas and alleviate social problems, a series of land reclamation projects or new town projects have been launched. However, since the enactment of the Protection of the Harbour Ordinance in 1997, environmental problems associated with land reclamation have been a concern and land reclamation has dropped around 80% (CEDD, 2022). To accommodate more citizens and growing urbanization, the development of more land is initiated again in recent years. In particular, the feasibility of the East Lantau Metropolis or Lantau Tomorrow, has been evaluated recently. A regional geological map of East Lantau is shown in Figure 1, which mainly consists of rhyolite dykes and granite. Still, land reclamation requires thorough ground investigations, assessment of related risks and satisfaction of sustainability principles. To simultaneously reach the objectives of sustainability and minimising the risks resulted from the lack of understanding of subsurface conditions, it is crucial to understand the seabed conditions before conducting land reclamation (Dong et al., 2014; Hack et al., 2005).

A huge number of reports from ground investigations has been obtained over the years in Hong Kong, including borehole data and results of laboratory tests. If these data are processed and managed properly, it will be valuable in preventing or mitigating ground risks at an early stage, and facilitating project management by a robust and realistic evaluation of project proposal and costs.

It is understood that advanced computations empower faster data collection, management and modelling in multi-dimensions (Aleksandrov et al., 2019; Balsa-Barreiro and Fritsch, 2018). Taking advantage of technological advances, 3-D geological visualisation is ubiquitous around the world (Self



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et al., 2012; Pan et al., 2018). However, the engineering community in Hong Kong mainly uses 2-D maps (Xiao et al., 2017), such as borehole sections, geological profiles and plans. It is rare for engineers or geologists to visualise in 3-D format or construct 3-D geological models using common modelling platforms, such as RockWorks, Leapfrog, GOCAD, and Civil 3D. These platforms are not very satisfactory for large-scale, complicated geological modelling. Hence, it is beneficial and essential to develop user-friendly toolboxes involving data management and 3-D geological modelling for Hong Kong. 3-D modelling and visualisation enables better understanding of the geological environment because of direct representation of data in 3-D space, which is more exact, direct and dynamic (Shao et al., 2011). Not only 3-D modelling provides innovation to geological and geotechnical study, it also analyses the geological formation, geological structures, attributes associated with geology and geotechnical parameters (Gong et al., 2004; Collon et al., 2015; Wang et al., 2015). Therefore, the 3-D modelling technique will bring significant changes in geological data management and display, providing a scientific basis and technical support for engineering decisions (Shao et al., 2011; Wu and Xu, 2004).

Developing methodologies, proposing standards of 3-D geological model and facilitating the ability of decision making on geo-environmental implications hence become the driving force of this study. Several digital databases, including buildings, road network, drainage system, land use planning, marine traffic density, marine resources, terrain elevation and seabed level, marine ecological system, geohazards, geology and ground investigation records, are collected (Figure 1), with the help of several governmental departments, such as Civil Engineering and Development Department, Lands Department, and Marine Department. The development of the geological modelling and management system is presented, taking a 3-D marine geological model of East Lantau as an example. Further possibilities and limitations of the developed analytical toolbox and 3-D modelling capabilities are evaluated in this study.

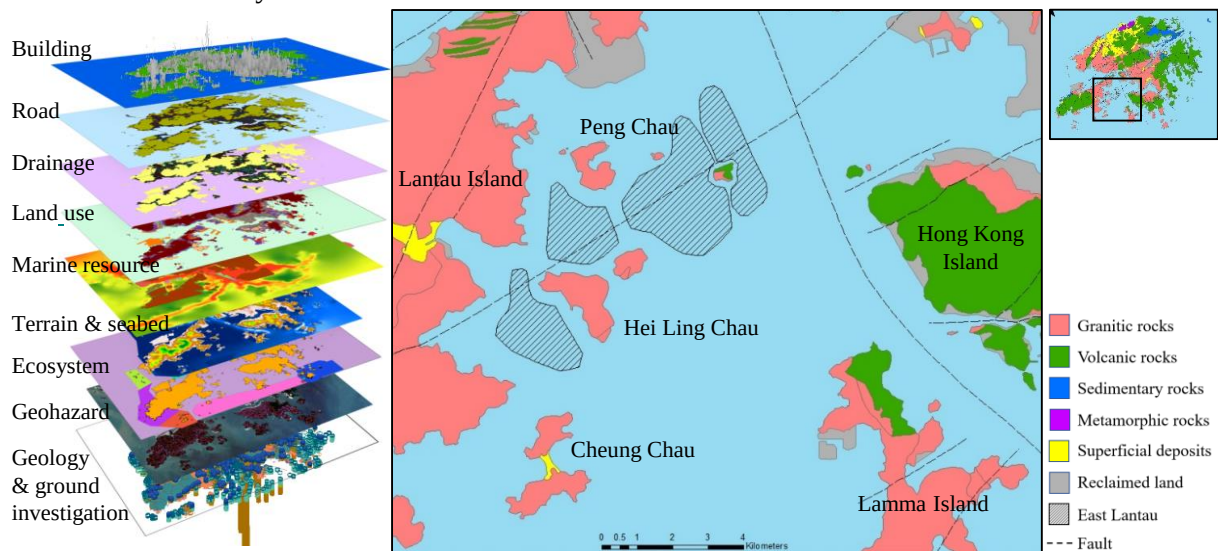


Figure 1: Hong Kong geo-databases and digitalised geology for East Lantau

2 Development of 3-D Geological Modelling and Management System

The 3-D geological modelling and management system is developed for entire Hong Kong. East Lantau is presented as a case study to demonstrate the functions of the system. The methodology is implemented in three phases (Figure 2), including data preparation, development of associated toolboxes and 3-D geological model construction. All the development is based on ESRI ArcGIS Pro which provides various geoprocessing tools for 2-D and 3-D interactions and supports Python programming and web-based data visualisation and sharing, although it is not originally designed for

3-D geological modelling. Considering that ArcGIS platform has been widely used in the industry, it is the most suitable platform for the development of the geological modelling and management system.

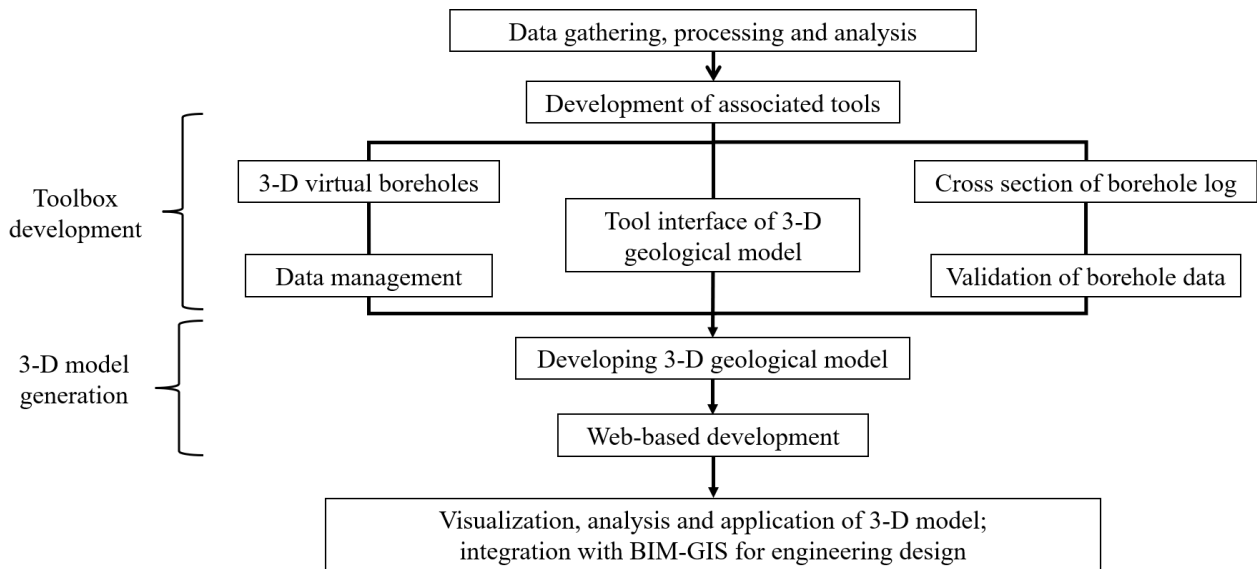


Figure 2: Methodology for developing a 3-D geological model

2.1 Borehole Data Preparation

We first digitalise available borehole data in AGS files and ground investigation reports from the Geotechnical Engineering Office, and conduct data clearance by correcting errors in coordinates, depth and material type and removing repeated records. Data verification is then performed by checking the conformity between borehole logs and geological maps, and between borehole collars and topographic maps. A total of approximately 90,000 boreholes with 667,000 records is finally verified in the entire Hong Kong, of which East Lantau has about 780 marine boreholes with 4,900 records, and converted into a “.csv” file for importing to ArcGIS. A verified record consists of location ID, report ID, hole ID, coordination, ground level, final depth, top depth, bottom depth, and geology code, as shown in Figure 3. Top and bottom elevations of each record in the toolbox are obtained by subtracting top depth and bottom depth from the ground level, respectively.

	A	B	C	D	E	F	G	H	I	J	Y	Z
1	Location ID	Report No	Hole ID	Location Type	Easting	Northing	Ground Le	Final Dept	Depth Top	Depth Bas	Geology Code	Geology Code 2
2	R_19722 H_BH-02	R_19722	H_BH-02	CP+RO+RC	821926.48	815848.66	-1.9	21.93	0	4.95	SAND	Marine deposit
3	R_19722 H_BH-02	R_19722	H_BH-02	CP+RO+RC	821926.48	815848.66	-1.9	21.93	4.95	9	CLAY	Marine deposit
4	R_19722 H_BH-02	R_19722	H_BH-02	CP+RO+RC	821926.48	815848.66	-1.9	21.93	9	13	SAND	Marine deposit
5	R_19722 H_BH-02	R_19722	H_BH-02	CP+RO+RC	821926.48	815848.66	-1.9	21.93	13	14.5	SAND	Alluvium
6	R_19722 H_BH-02	R_19722	H_BH-02	CP+RO+RC	821926.48	815848.66	-1.9	21.93	14.5	17.9	SAND	Grade V-IV rocks
7	R_19722 H_BH-02	R_19722	H_BH-02	CP+RO+RC	821926.48	815848.66	-1.9	21.93	17.9	20.23	GRANITE	Grade III-I rocks
8	R_19722 H_BH-02	R_19722	H_BH-02	CP+RO+RC	821926.48	815848.66	-1.9	21.93	20.23	21.93	GRANITE	Grade III-I rocks

Figure 3: Examples of borehole records in an Excel file

Figure 4 presents the frequency of 12 geological types in the borehole records of East Lantau, including N/A, fill, beach deposit, marine deposit, estuarine deposit, debris flow deposit, alluvium, colluvium, residual soil, grade V-IV rock, grade III-I rock and others. During modelling, the non-informative N/A and others are directly removed. Some geological groups, such as beach deposit, debris flow deposit, estuarine deposit, colluvium and residual soil, account for only small percentages comparing with the dominant groups, such as marine deposit, alluvium and grade V-IV rock. They are combined with the dominant group according to the stratigraphic property, forming five main groups, namely fill, marine deposit, alluvium, grade V-IV rock and grade III-I rock.

In addition, each borehole record has different lengths (for example, the first record in Figure 3 is 4.95 m and the fourth record is 1.5 m) and they should take different weights in geological modelling.

For this purpose, every section of borehole log (i.e., a record in Figure 3) is discretised into a series of points with the same geological type. For example, if vertical resolution is 1 m, the first and forth records will be discretised as 5 and 2 points.

The geo-databases in Figure 1, such as territory boundary and seabed level, are useful in shaping the 3-D geological model into the region of interest and controlling the groundwater level of the model. When constructing the whole geological model for Hong Kong, terrain elevation is helpful in defining the boundary between mountain and air. Note that geological structures are not explicitly modelled in the current study, and they will be considered in the future.

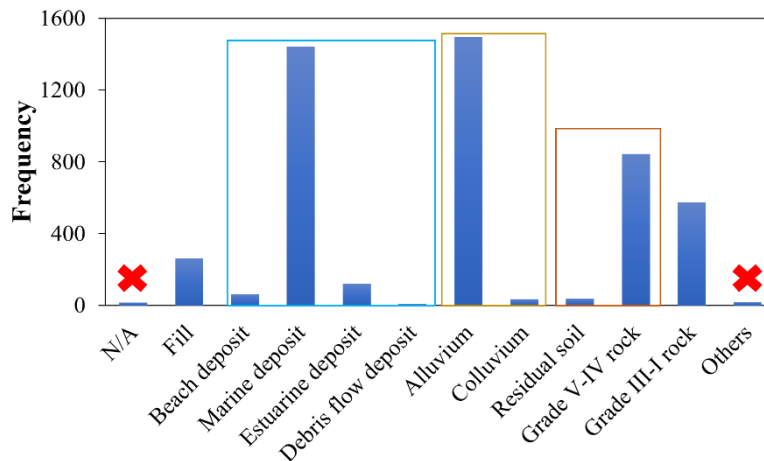


Figure 4: Distribution of geological type in borehole data

2.2 Toolbox Development

This study develops three ArcGIS toolboxes through Python programming, including 3-D virtual borehole toolbox, cross-section profiling toolbox and 3-D geological modelling toolbox, to enhance the capability of ArcGIS Pro in geological modelling and management. It is also flexible for users familiar with basic Python skills to customize the toolboxes.

2.2.1 3-D Virtual Borehole

This tool helps to manage borehole data systematically and visually in 3-D space. The primary principle is to make use of “`arcpy.Array`” and “`arcpy.Polyline`” in ArcGIS Pro to form 3-D polylines of boreholes by connecting 3-D points of records. The first step is to convey the delimited files containing borehole data in “.csv” to dBASE table by “`arcpy.conversion.TableToTable`” so that the borehole data is easier to be managed and searched in ArcGIS Pro as a geodatabase. Linking the top elevation and bottom elevation point together of each row from the borehole log table with functions of “`arcpy.SearchCursor`” and “`arcpy.UpdateCursor`” enables 3-D virtual borehole visualisation.

2.2.2 Cross-section Profiling

This tool allows users to focus on geological profile of interest by generating a 2-D cross-section of borehole log, elevation profile and groundwater level. The first phase is to obtain Z values and M values along the cross-section line using the function of “`arcpy.StackProfile_3d`”, where Z defines the elevation value along the cross-section line while M implies the measured distance from the beginning of the cross-section line (Figure 5). With the points at the beginning and at the end of the cross-section line, coordinates of each borehole log (existing x and y value) could be converted into M values by applying the Euclidean distance. Top and bottom elevations are transformed into Z values simultaneously by ensuring conversion under the same Borehole ID to avoid mixing up borehole logs.

Horizontal and vertical exaggerations are optional if users would like to expand or squeeze the section view. It is suggested that users compress the horizontal distance by inputting a number smaller

than 1 as the horizontal distance is usually much greater than the vertical distance. The temporary feature class output is a cross-section polyline profile. With the temporary polyline feature and function of “arcpy.analysis.Buffer”, borehole logs in the cross-section view hence could be constructed as polygons, together with the polyline feature of terrain.

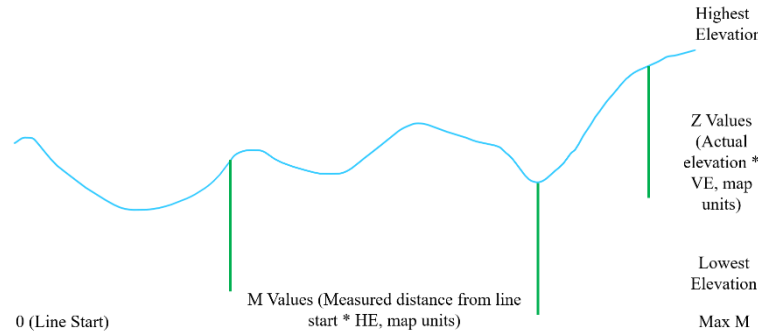


Figure 5: A surface profile and borehole log using M and Z values obtained

2.2.3 3-D Geological Modelling

This tool aims to provide automatic modelling of 3-D geological conditions in a region defined by users, lessening the coding requirements. The first step is to set up an interface for users to define the boundary where they would like to model. A 3-D geological modelling algorithm is then selected with necessary model parameters. The output file extension is “.nc”, Network Common Data Form (NetCDF), which supports the storage of 3-D array data (Unidata, 2022). With a designated output path and file name, the 3-D geological model could be added and viewed in ArcGIS Pro.

This study makes use of three machine learning algorithms including k-nearest neighbours (kNN), support vector machine (SVM) and random forest (RF) (scikit-learn, 2022) for 3-D geological modelling. In the toolbox, parameters of each algorithm are set at default values, in which the number of neighbours used in kNN is 15, the kernel coefficient gamma of SVM is 0.5 and the number of trees in RF is 45. The borehole data is randomly divided into 7:3 for training and test sets. A unified parameter, anisotropy ratio δ , is introduced to eliminate the anisotropic effect between the horizontal scale and vertical scale in geological setting, as:

$$x' = \frac{x}{\delta} \quad (1)$$

where x = the horizontal coordinates and x' = the transformed horizontal coordinates.

The F-1 score is used for model comparison, which is defined as a harmonic mean of the precision and recall of model performance. The precision accounts for the ratio of true positive outcomes to all the positive outcomes, and the recall means the ratio between the true positive results and the number of results that should have been positively identified. Specifically, the F-1 score is calculated as follows:

$$F1 = \frac{2(\text{precision} \times \text{recall})}{(\text{precision} + \text{recall})} \quad (2)$$

F-1 score value of 1 is the best while 0 is the worst performance, reflecting the goodness of the interpolated geological model.

In addition to the 3-D geological modelling, the information entropy at each location is also estimated to measure the uncertainty of the interpolated model (Zhao and Wang, 2019; Xiao et al., 2021). The entropy shows the level of “surprise” with respect to the possible results, and is defined as:

$$H(Y) = -\sum_{i=1}^n P(y_i) \log P(y_i) \quad (3)$$

where $P(y_i)$ = probability of possible geological condition, estimated from the machine learning algorithms; y_i = possible geological condition (i.e., fill, marine deposit, alluvium, grade V-IV rock and grade III-I rock); n = number of possible geological conditions (i.e., 5 in this study). A larger information entropy means a higher geological modelling uncertainty.

3 Case Study of East Lantau

East Lantau, one of the major future developments in Hong Kong, is taken as a case study. The selected region is between 820,000 and 830,000 Easting, and between 810,000 and 819,000 Northing (Figure 6). Within the selected region, about 780 marine boreholes with average drillhole depth of 33 m and an average water depth of 6 m are employed for demonstration. After combining the stratigraphy types as mentioned in section 2.1, the ratio of fill: marine deposit: alluvium: grade V-IV rock: grade III-I rock is 5:33:32:18:12, showing that marine deposit and alluvium are the top two strata in East Lantau.

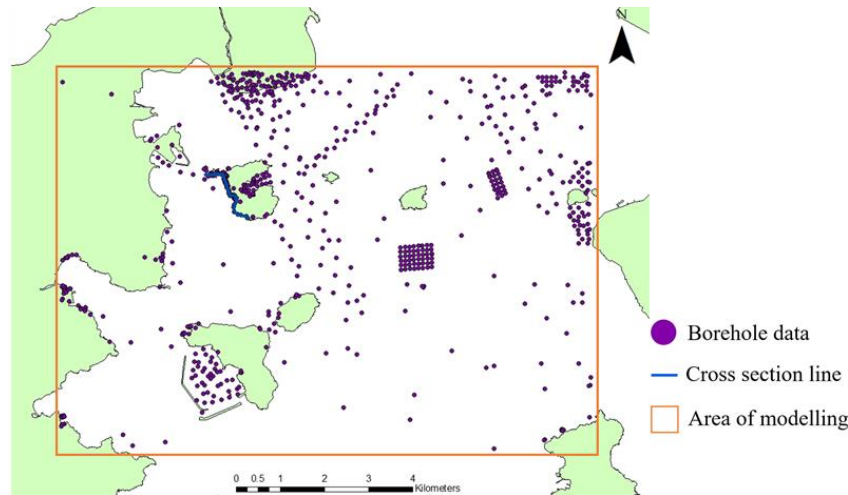


Figure 6: Boreholes in East Lantau

3.1 3-D Borehole Data Management

The interface of the 3-D virtual borehole toolbox is illustrated in Figure 7(a). This tool creates 3-D virtual drillholes from the borehole log table, offering easier borehole data management and searching functions of the borehole information, as shown in Figure 7(b). If users would like to review boreholes information that they are interested in, they can click the created 3-D drillholes to view the details, as illustrated in Figure 7(c). The soil or stratigraphic layer is symbolized by a different colour, supporting another method of validation on the 3-D geological model.

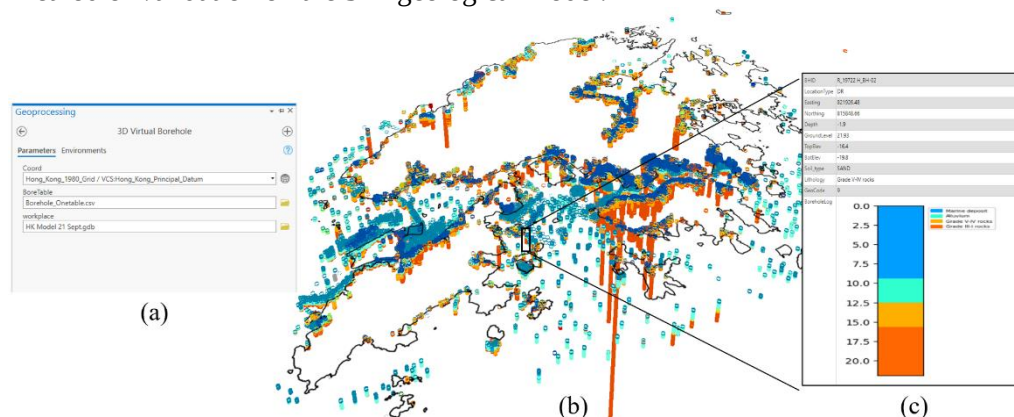


Figure 7: 3-D virtual boreholes: (a) toolbox interface; (b) all digitalised boreholes; (c) details of one borehole

3.2 Cross-section of Borehole Logs

The interface of the cross-section profiling toolbox is shown in Figure 8(a), which chooses borehole data within 100 m along the west Peng Chau (i.e., the cross-section line in Figure 6) to produce borehole logs with 10 times vertical exaggeration and 0.5 times horizontal exaggeration. Figure 8(b) presents the cross section, in which the reference elevation profile is obtained from the digital terrain and seabed

elevation model and boreholes are selected within the specified searching radius. It is important to accurately trace the position of each exploration and the related borehole log along the cross-section line. The thickness of each layer in each borehole is depicted, which is derived from the distance between the top and bottom elevation field.

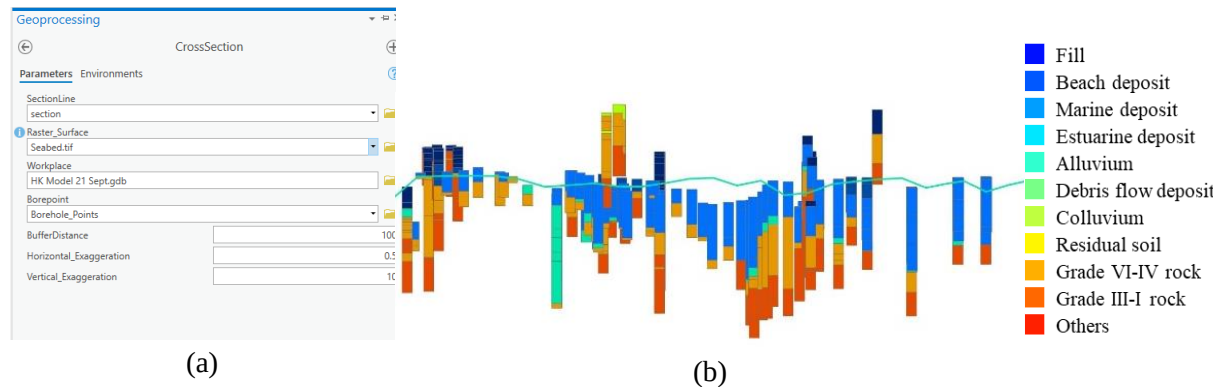


Figure 8: Cross-section of borehole logs: (a) toolbox interface; (b) an example along the west Peng Chau

3.3 3-D Geological Model and Associated Modelling Uncertainty

The toolbox interface of the 3-D geological model is shown in Figure 9(a), allowing users to choose the boundary that they are interested in and a specified machine learning algorithm (available algorithms: kNN, SVM, and RF). Figure 9(b) show a 3-D geological model with space resolution of 50 m by 50 m by 1 m and the fence diagram of the East Lantau generated from kNN with an anisotropy ratio of 100 and a neighbour size of 15. Anisotropy ratio refers to the scale weight difference between the horizontal scale and vertical scale. Positions of the faces can also be adjusted to locations of interest using sliders in ArcGIS Pro when users need to focus on specified subsurface sections.

This toolbox also measures the modelling uncertainty through information entropy. Figure 9(c) demonstrates the information entropy of the geological model in Figure 9(b). The uncertainty of bedrock is much smaller than other stratigraphic layers. This is reasonable since the unknown points in deeper elevation are predicted from observed points that mostly belong to grade III-I rock. The zone between Lantau Island and Hei Ling Chau, the area around Peng Chau and the open area of the East of Hong Kong Island have higher uncertainties. Hence, more detailed site investigation works could be considered on these areas, bringing about higher affirmation on the subsurface conditions.

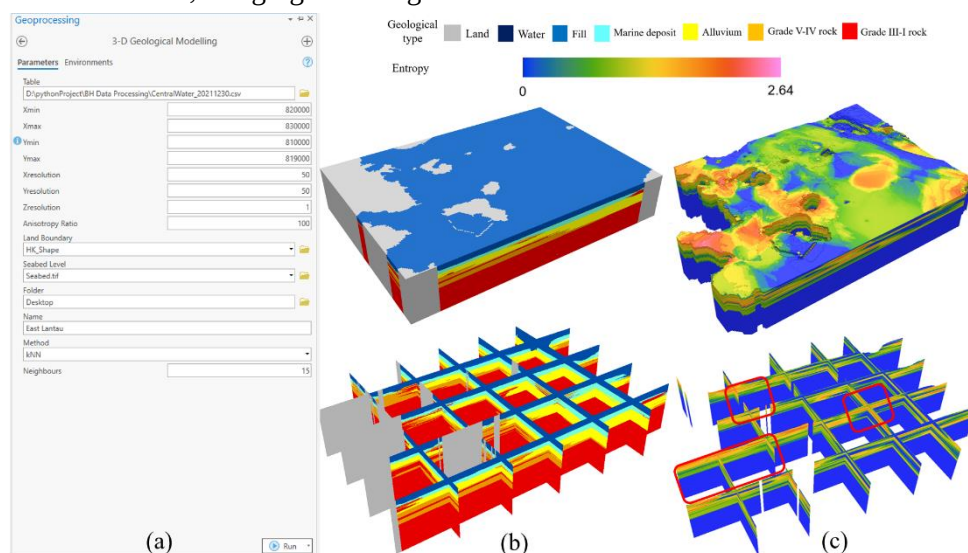


Figure 9: 3-D geological model: (a) toolbox interface; (b) a 3-D model of East Lantau using kNN; (c) associated information entropy of the 3-D model

4 Impact of Anisotropy Ratio in Geological Modelling

This section compares the geological models of East Lantau generated using all the three implemented algorithms (i.e., kNN, SVM, and RF) with five typical anisotropy ratios (i.e., $\delta = 1, 10, 100, 1000$, and 10000 in Eq. (1)). Figure 10 provides their F-1 scores for both training and test sets. Note that a weighted average of F-1 score for each class is applied to avoid the effect of the imbalance classification. The F-1 score of kNN and RF algorithms for the training set at different anisotropy ratio is highly overlapped at 0.95. Meanwhile, the F-1 score of kNN for the test set reaches the maximum value (i.e., 0.7) at an anisotropy ratio δ of 100. Such an anisotropy ratio means that two borehole points with horizontal separation distance of 100 m is equivalent to two borehole points with vertical separation distance of 1 m. This is similar to many studies on characterising spatial variability of soils, which found that the horizontal scale of fluctuation is about one or two orders of magnitude larger than the vertical scale of fluctuation (Phoon and Kulhawy, 1999).

Figure 11 further compares the detailed geological profiles of west Peng Chau using kNN with three anisotropy ratios. The original borehole logs are shown in Figure 11(a) for reference. Visually, when δ is less than 100 (Figure 11(b)), the estimation of stratigraphy is more dominated by vertical separation of observed samples, and the information from nearby boreholes (e.g., 100 m away) are not fully used. On the other hand, if δ is greater than 100 (Figure 11(d)), the horizontal separations provide higher importance than the vertical separations, which undermines the nature of the geological formation and leads to the overfitting of stratigraphy. As a result, to prevent modelling from wrongly predicted by the dominance of vertical separations and overfitting from horizontal separations, an optimal anisotropy ratio δ should be determined during the construction of a 3-D geological model and $\delta = 100$ appears most reasonable in this case study (Figure 11(c)).

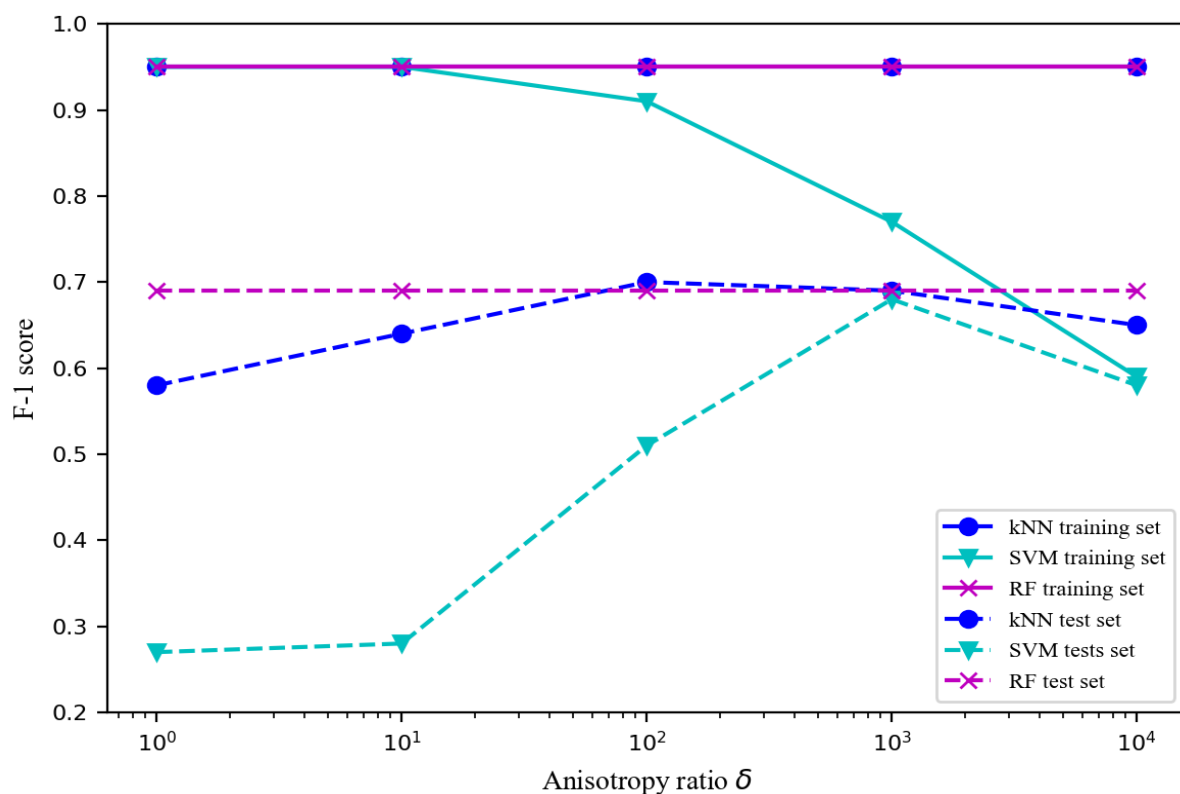


Figure 10: Comparison of weighted average of F-1 score among algorithms

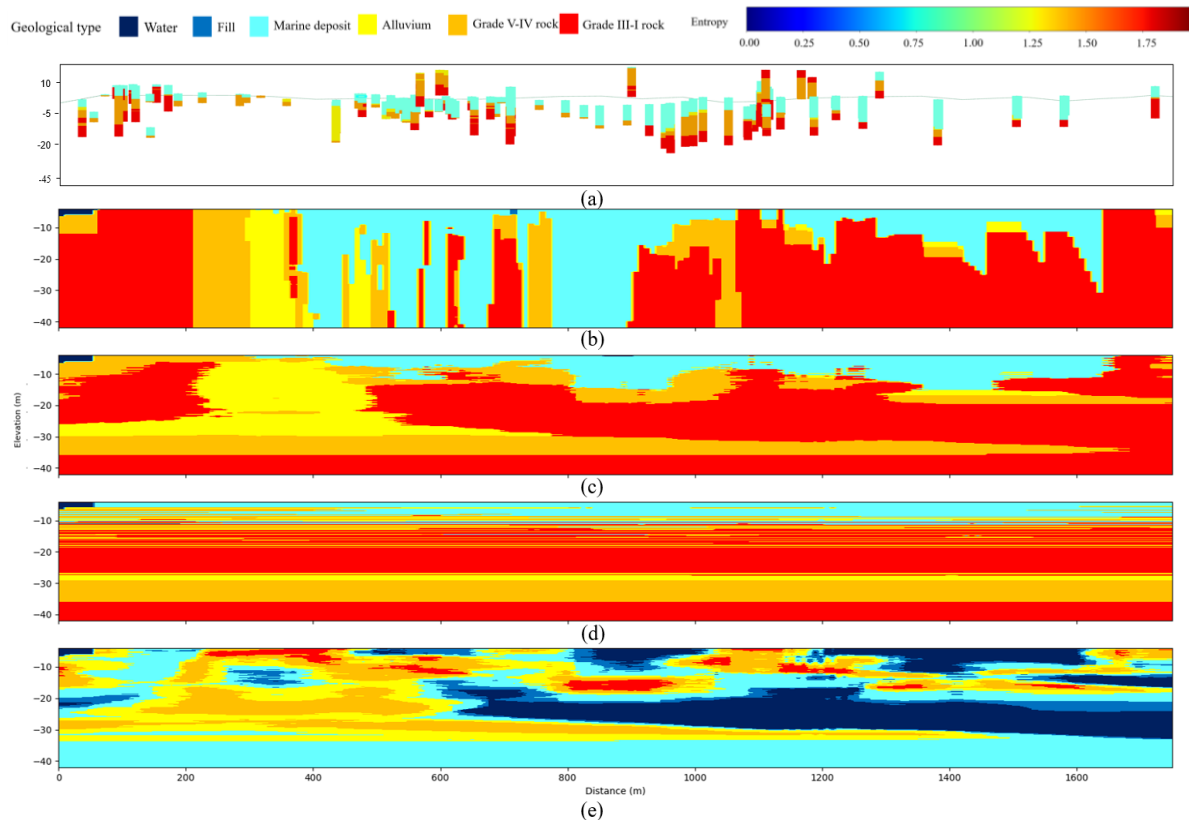


Figure 11: Example of west Ping Chau: (a) original borehole log; (b)-(d) predicted geological profiles using kNN with anisotropy ratio $\delta = 1$, 100, and 10,000, respectively; (e) information entropy of the geological model with $\delta = 100$

5 Conclusions and Recommendations

A set of ArcGIS tools have been developed for territory-wide borehole data management and 3-D geological model visualisation in Hong Kong. East Lantau is used as an example to illustrate the results of toolboxes and 3-D geological model construction.

Data clearance is required to eliminate duplicates or typographical errors before applying the toolbox and building the 3-D geological model. The 3-D borehole log and cross-section borehole log further provide visualised tools to validate the borehole data, enhancing the quality and quantity of the input data.

The 3-D geological model is established by machine learning algorithms. An anisotropy ratio is introduced to eliminate the anisotropic effect between the horizontal scale and vertical scale in geological setting. Information entropy is provided to quantify the uncertainty of the model. With the aid of fence diagrams or cross sections obtained from the 3-D model, geologists or engineers can have a better understanding of the local geological conditions and the complex subsurface profiles. In addition, it is reminded that a geological model is a tool for conducting a more effective site investigation work instead of substitution. Whenever new borehole data is acquired from site investigation, they in turn can be employed to verify or improve the geological model so that the reliability of subsequent geotechnical or structural design can be enhanced due to a great reduction in uncertainties.

Web-based ArcGIS map service can be further developed in the future so that the public can access and view the borehole data or 3-D geological model and use the developed toolboxes without installing ArcGIS platforms, providing much smoother interactions among engineers, governmental agencies and the public.

6 Declarations

6.1 Acknowledgements

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