A Comparison of Empirical and Numerical Approaches for Estimating Rock Support Pressure on Permanent Tunnel Lining

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Abstract

In Hong Kong, the rock support pressure acting on the permanent tunnel lining is usually estimated using the empirical equations by Terzaghi's rock arching theory (1946) and Grimstad & Barton's Q support pressure (1993). However, with the advanced technologies, the assumptions behind these studies may become too conservative and subsequently lead to high construction cost and time. According to the Geoguide 4 (2018 Edition), it is suggested that the rock support pressure should be estimated either by an empirical method or an analytical/numerical assessment. By establishing different comparison models, this paper investigates the difference in estimated rock support pressure acting on the permanent lining using empirical approaches and finite element modelling. The influence of missing parameters in empirical equations and the rock mass behavior around the excavation profile are also studied.

Keywords: Rock tunnel, Permanent lining, Empirical approach, Numerical modelling

1 Introduction

Only less than 25% of total land area in Hong Kong has been developed for the 7.5 million population. This is because the rest settings have been surrounded by hilly terrains, rural areas and statutory protected areas. Nevertheless, significant portions of these hilly terrain features are underlain by hard and massive igneous rocks such as granite and volcanic tuff. This brings favorable conditions to develop underground space such as rock tunnels and caverns as an alternative source for land supply. In recent years, the government has been playing a leading role to explore the feasibility and strategy for long-term underground development in Hong Kong.

In consideration of the consequence of life, durability and maintenance problems, temporary supports such as rock dowels and shotcrete are usually not taken to contribute any of the long-term ground stability. Most of the constructed rock tunnels in Hong Kong are permanently supported by conventional cast-in-situ concrete lining. If design optimization for lining (e.g. thickness and reinforcements) is achieved, the overall construction cost and time for excavating the rock, rebar fixing and casting the concrete lining will be significantly reduced.

From geotechnical design perspective, the estimation of rock support pressure acting on the permanent lining is very uncertain because it cannot be simply measured and well-proved. In Hong Kong, the rock support pressure has been estimated using the empirical equations by Terzaghi (1946) and Grimstad & Barton (1993) based on the back-analysis of installed support and field data collection. As suggested by GEO (2018), the source of loadings should be either accounted for by an empirical method or an analytical/numerical assessment. By establishing different comparison models, this paper investigates the difference in estimated rock support pressure on lining using empirical

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approaches and finite element modelling. The influence of missing parameters in empirical equations and the rock mass behavior around the excavation profile are also studied.

2 Rock Tunnelling with Empirical Approaches

2.1 Terzaghi's Rock Arching Theory (1946)

The rock arching theory developed by Terzaghi in 1946 was the first successful rock mass classification for tunnel engineering. The theoretical rock arch above crown is self-supporting and only the weight of loosened rock after excavation is acting on the tunnel supports. The support pressure was estimated based on the known strength of failed wooden blocks and back-analysis of a 5.5m wide tunnel supported by steel-arches.

He defined the term "arch action", which indicates the capacity of rock above the tunnel roof to transfer the major part of the total overburden weight to both sides of tunnel walls. The body of rock which transfers the load will briefly be referred to as the ground arch. The arch action is an inevitable consequence of the local stress relaxation produced by excavation operation. The mechanics of the arch action is illustrated in Figure 1. The ground arch is represented by the shaded area **a c d b** with a height of D and width of B_1. When the tunnel is being excavated and the support installed, the mass of crushed rock constituting the ground arch tends to move into the tunnel. This movement is resisted by the friction along the lateral boundaries **a c** and **b d** of this mass. The friction forces transfer the major part of the total overburden weight with height H onto the material located on both sides of the tunnel and the roof support carries only the balance, equivalent to a height $D=H_p$.

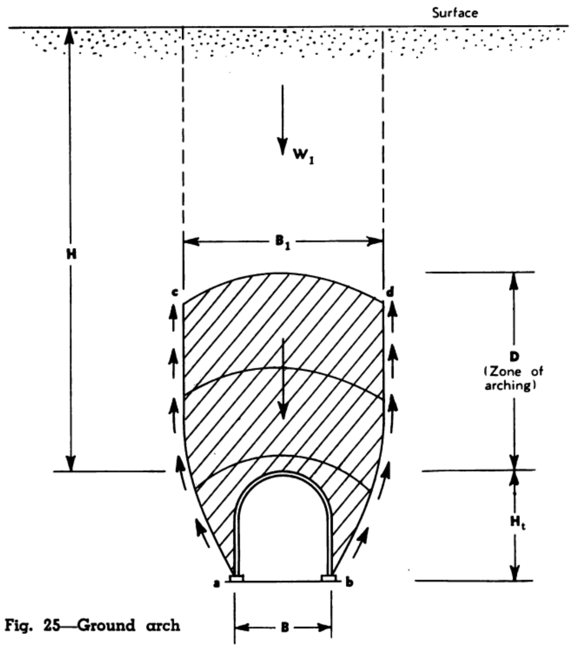

Figure 1: Ground Arch (Terzaghi, 1946)

Terzaghi combined his experiment results and the estimated rock loads from Alpine tunnels to compute rock load factors H_p of the loosened rock mass above the tunnel crown (as listed in Table 1) in terms of tunnel width B and tunnel height H_t. The vertical support pressure (P_v) is defined by Equation 1:

$$P_v = g \times D = g \times H_p \qquad (1)$$

Table 1. Rock load in tunnels within different nine rock classes (Terzaghi, 1946)

Rock Condition	**Rock Load H_p ***	**Remark**
1. Hard and intact	Zero	Light lining required only if spalling or popping occurs
2. Hard stratified or schistose	0 to 0.5B	Light support.
3. Massive, moderately jointed ^	0 to 0.25B ^	Load may change erratically from point to point.
4. Moderately blocky and seamy	0.25B to 0.35 (B + H_t)	No side pressure
5. Very blocky and seamy	0.35 to 1.10 (B + H_t)	Little or no side pressure
6. Completely crushed but chemically intact	1.10 (B + H_t)	Considerable side pressure. Softening effects of seepage toward bottom of tunnel requires either continuous support for lower ends of ribs or circular ribs.
7. Squeezing rock – moderate depth	1.10 to 2.10 (B + H_t)	Heavy side pressure, invert struts required. Circular ribs are recommended.
8. Squeezing rock – great depth	2.10 to 4.50 (B + H_t)	
9. Swelling rock	Up to 250 ft (80 m), irrespective of the value of (B + H_t)	Circular ribs required. In extreme cases use yielding support.

Note:

* *The roof of the tunnel is assumed to be located below the water table. If it is located permanently above the water table, the values given for types 4 to 6 can be reduced by fifty percent. However, this pressure reduction has not been considered in the study.*

^ Rock Load H_p=0.25B is adopted for this study where the baselined Q-value is 10.

2.2 Grimstad & Barton's Q Support Pressure (1993)

The empirical Q-system for rock mass classification and its relationships to tunnel supports were first developed by Barton et al. (1974) and further updated by Grimstad & Barton (1993) at the Norwegian Geotechnical Institute (NGI), Norway by considering more than 1000 case histories from underground openings. The tunnel support pressure at crown (P_{roof}) and walls (P_{wall}) for different rock mass conditions are estimated using Equation 2 and Equation 3.

$$P_{roof} = \frac{0.2\, J_n^{1/2}\, Q^{-1/3}}{3\, J_r} \quad \text{(MPa)} \qquad (2)$$

$$P_{wall} = \frac{0.2\, J_n^{1/2}\, Q_w^{-1/3}}{3\, J_r} \quad \text{(MPa)} \qquad (3)$$

where Q = Tunnelling Quality Index
J_n = Joint Set Number
J_r = Joint Roughness Number

Q_w = 5.0 Q for Q > 10
2.5 Q for 10 ≥ Q ≥ 0.1
1.0 Q for Q < 0.1

2.3 Missing Parameters in Empirical Approaches

For a tunnel project, there are two major types of design parameters including the geotechnical parameters and tunnel arrangements. The considerations of these variables to the rock support pressure by each empirical study were very limited based on their specific engineering assumptions and field data as summarized in Table 2. Both studies have incorporated the importance of rock mass quality and only Terzaghi (1946) has considered the significance of tunnel width. However, none of these two approaches have considered the influence of varying uniaxial compressive strength, intact rock modulus, poisson's ratio, in-situ stress ratio and tunnel depth to the rock support pressure. Therefore, it is important to study whether engineers have been missing any governing parameters to estimate the rock support pressure throughout the years.

Table 2. Summary Table of Design Parameters Considered in Previous Empirical Approaches

Empirical Studies/ Rock Parameters	**Uniaxial Compressive Strength $\sigma_{\chi\iota}$**	**Intact Rock Modulus E_i**	**Poisson's Ratio *v***	**In-situ Stress Ratio *k***	**Rock Mass Quality**	**Tunnel Depth**	**Tunnel Width**
Terzaghi (1946)					✓		✓
Grimstad & Barton (1993)					✓		

3 Rock Tunnelling with Numerical Modelling

3.1 Setting up the Comparison Models

Apart from empirical approaches, the rock support pressure can also be estimated using the numerical modelling such as the finite element method (FEM) and the discrete element method (DEM). They are powerful tools to handle complicate engineering problems such as complex geology, imposed loadings, influence from/to existing buildings and structures, excavation sequence, composite structures and 3-dimentional geometric problems.

There were total 57 numbers of finite element models established using the software Phase^2 Version 8 developed by the Rocscience (BD Ref: G0179) for this study. The baseline geotechnical parameters and tunnel arrangements were adopted based on a highway tunnel project in Hong Kong. The excavation span (or tunnel width, *D*) was 13.5 m and the tunnel depth was approximately 110 m (8 times the tunnel width *D*) below ground level.

3.2 Rock Mass Failure Criteria

The strength of a jointed rock mass depends on both the mechanical properties of intact rock as well as the degree of freedom for the rock block to slide and rotate under different stress states. The yield mechanism is non-linear and the failure mechanisms are often brittle.

The Hoek-Brown Failure Criterion was first derived by Hoek and Brown (1980) from testing results of rock specimen to estimate the deformation and strength characteristic of jointed rock mass based on the interlocking effect and discontinuities conditions. Later in 2002, the modified Generalized Hoek-

Brown (GHB) Failure Criterion was further developed as presented in Equations 4 to 7 to overcome the bias of data towards hard rock.

$$\sigma'_1 = \sigma'_3 + \sigma'_{ci}\left(m_b \frac{\sigma'_3}{\sigma'_{ci}} + s\right)^a \quad (4)$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \quad (5)$$

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \quad (6)$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right) \quad (7)$$

where σ'_1 = Major Effective Principal Stress
σ'_3 = Minor Effective Principal Stress
σ_{ci} = Uniaxial Compressive Strength
m_i = Hoek-Brown Intact Constant
m_b = Hoek-Brown Constant
s = Rock Mass Materials Constant
a = Rock Mass Materials Constant
D = Blast Disturbance Factor

The Geological Strength Index (GSI) was introduced by Hoek (1994) and Hoek, Kaiser and Bawden (1995). It is used to estimate the rock mass strength by considering the reduction of the intact rock strength due to adverse rock structure and block surface conditions. It can be corelated to the NGI Q-system using Equation 8.

$$GSI = 9 \ln Q' + 44 \quad (8)$$

where $Q' = \frac{RQD}{J_n} \times \frac{J_r}{J_a}$

3.3 Geotechnical Parameters and Tunnel Arrangements

For each of the comparison model, there was only one varying parameter and the remaining numerical inputs were assigned in accordance with the baseline assumptions. These parameters consist of the geotechnical parameters and tunnel arrangements as summarized in Table 3 and were assumed based on a highway tunnel project in Hong Kong. The varying parameters in Table 4 were assigned at appropriate intervals within the common design ranges by considering the geology conditions in Hong Kong.

Table 3. Summary Table of Baseline Parameters

Baseline Parameters	Baseline Value			Descriptions
Unit Weight	γ	=	27 kN/m^3	-
Uniaxial Compressive Strength	σ_{ci}	=	75 MPa	-
Intact Rock Modulus	E_i	=	30000 MPa	E_m estimated by Geoguide 4 Eq. 6.8
Poisson's Ratio	v	=	0.3	-
Tunnel Depth	Depth	=	8*D* = 108 m	Moderate Depth
Tunnel Width	*D*	=	13.5 m	Medium Span
Tunnel Height	H	=	9.5 m	-
Rock Quality Designation	RQD	=	80	Good
Joint Set Number	J_n	=	12	Three joint sets plus random joints
Joint Roughness Number	J_r	=	1.5	Rough, irregular, planar
Joint Alteration Number	J_a	=	1	Unaltered joint walls, surface staining only
Joint Water Reduction Factor	J_w	=	1	Dry excavations or minor inflow
Stress Reduction Factor	SRF	=	1	Medium stress, favorable stress condition
Q-value	Q	=	10	Fair/Good Rock
In-situ Stress Ratio	k	=	1.5	-
Material constants	m_i	=	32	Granite
Lining Thickness	T	=	350 mm	C40 plain concrete lining

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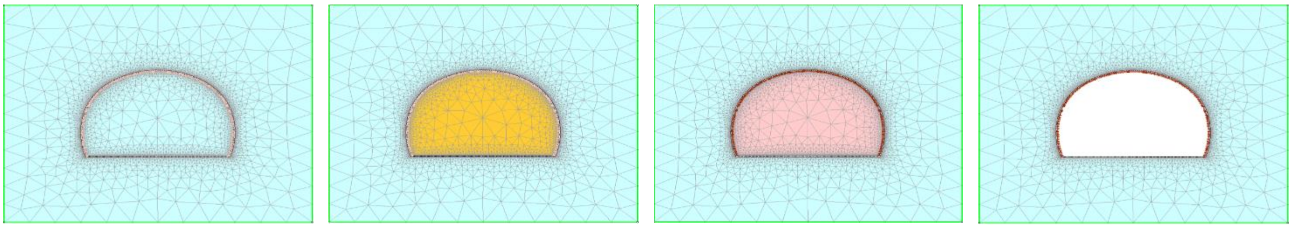
Table 4. Summary Table of Varying Parameters

Varying Parameters	Ranges
Uniaxial Compressive Strength σ_{ci}	50, 75, 100, 125, 150, 175, 200 MPa
Intact Rock Modulus E_i	20000, 25000, 30000, 35000, 40000, 45000, 50000 MPa
Poisson's Ratio *v*	0.2, 0.25, 0.3, 0.35, 0.4
In-situ Stress Ratio *k*	0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4
Q-value (assume Q=Q')	0.1, 0.4, 1, 4, 10, 25, 40
Tunnel Depth	1*D* to 15*D* (13.5 m to ~200 m), per *D* intervals
Tunnel Width *D*	5.5, 8, 13.5, 20, 30 m

3.4 Construction Sequence and Staged Analysis in FEM

There were 4 stages assigned to each of the finite element model as presented in Figure 2 for different construction sequence. Stage 1 calculated the original in-situ stress stage of the rock mass before excavation. Stage 2 simulated the ground relaxation with the core replacement method to consider the 3-dimensional effect of tunnel face supporting the surrounding rock mass. Stage 3 simulated further ground relaxation of rock mass due to advance excavation and the installation of permanent concrete lining support. Stage 4 modelled the complete excavation and all ground loadings were exerted to the permanent concrete lining.



Stage 1 Stage 2 Stage 3 Stage 4

Figure 2: Staged Analysis in FEM

3.5 Assumptions and Limitations

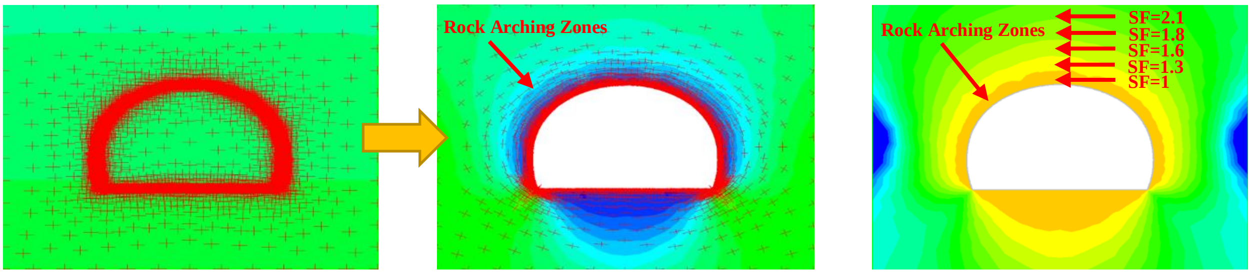
The FEM comparison models have the following assumptions and limitations:

- Single parametric study was carried out where each comparison model only changed one parameter. Therefore, some of the observations and statements made under this paper may only be appliable for the condition of only one varying parameter adopted
- Rock mass was modelled as continuum element with rock mass modulus E_m and GHB failure criterion instead of discontinuum element. Rock wedges should be considered separately using kinematic analysis
- In-situ stress was induced by gravity and the actual ground level in models
- No relaxation during temporary stage was considered. The permanent support was installed after tunnel excavation by assuming constant ground relaxation for varying rock mass quality and tunnel width
- Groundwater pressure should be considered separately
- Excellent and controlled blasting. No significant damage to surrounding rock mass.

4 Results and Discussions

4.1 Rock Arching after Tunnel Excavation

Different rock arching zones were identified in Figure 3 by observing the principal stress orientation and rock mass strength factor after the removal of the original rock mass (i.e. tunnel excavation). The orientation of major principal stress (s_1) rotated and became perpendicular to the excavated profile which provides the hoop force for rock arching. For the rock strength factor, which is calculated by dividing the rock strength (based on GHB failure criteria) by the induced stress at every point in the mesh, dropped significantly after excavation from about 4 to 1 near the excavation boundary and gradually increased away from the opening. These results from numerical modelling verified the rock arching theory proposed by Terzaghi in 1946.



Principal Stress Orientation (before and after excavation) Rock Mass Strength Factor

Figure 3: Rock Arching after Tunnel Excavation

Figure 4 shows the stress contours at different tunnel depths and Figure 5 presents the close-up views at depths of 1*D* (13.5 m), 4*D* (~55 m), 7*D* (~95 m), 8*D* (~110 m), 12*D* (~160 m) and 15*D* (~200 m). Similarly, the rock arching zones were observed in these comparison models. This implies that the permanent lining of a rock tunnel at moderate depth is only supporting the loosened rock mass after excavation instead of the full overburden pressure to the ground surface. However, a slow increasing trend of support pressure with increasing tunnel depth was observed by comparing the colour of stress contours around the excavation profile.

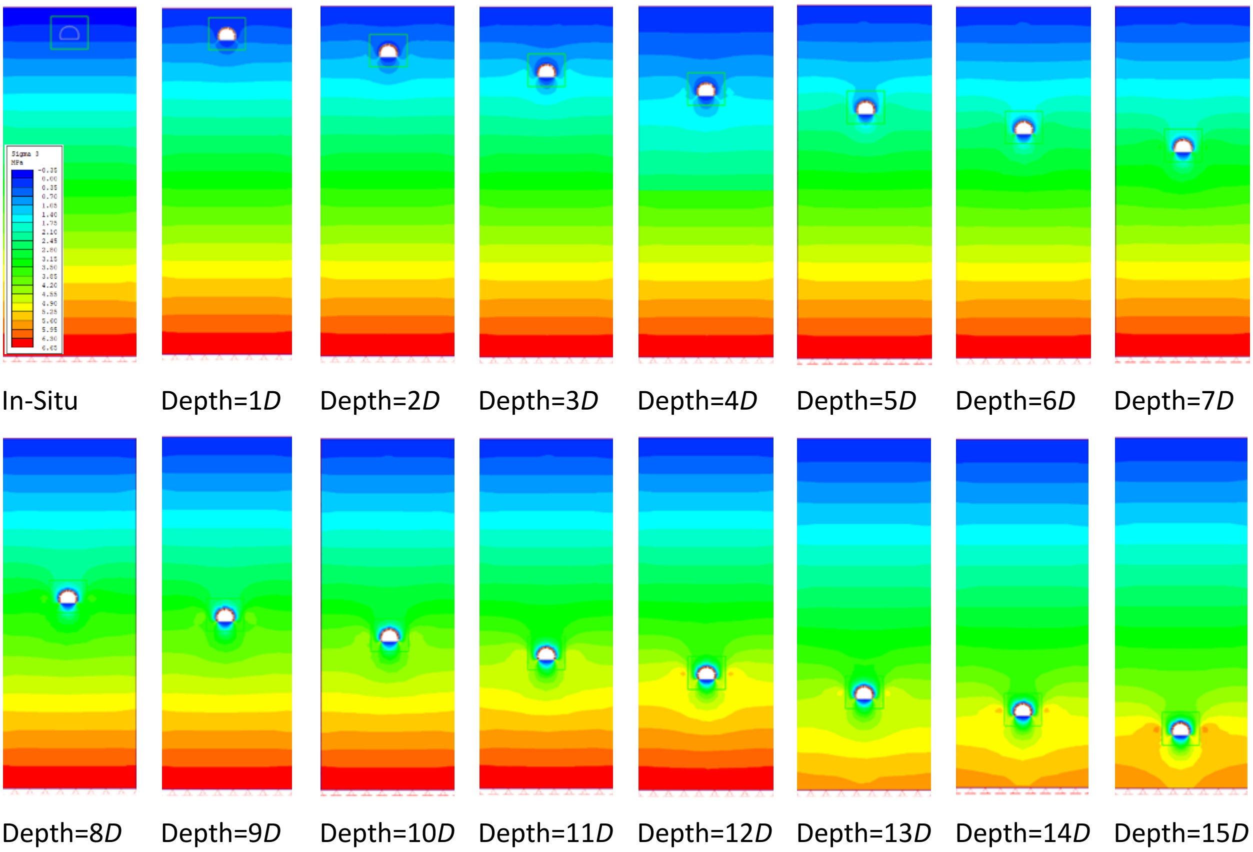


Figure 4: Stress Contours at different Tunnel Depths

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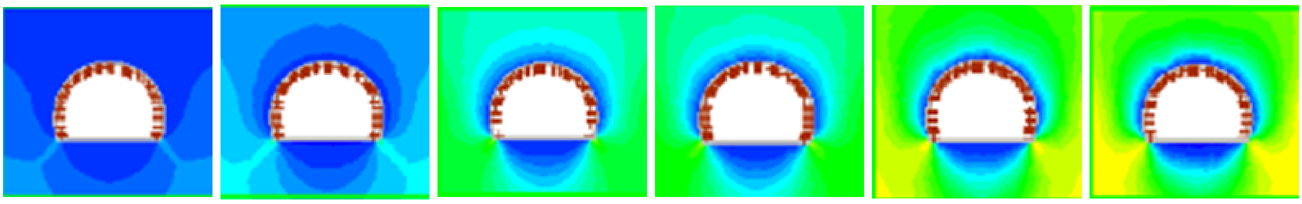


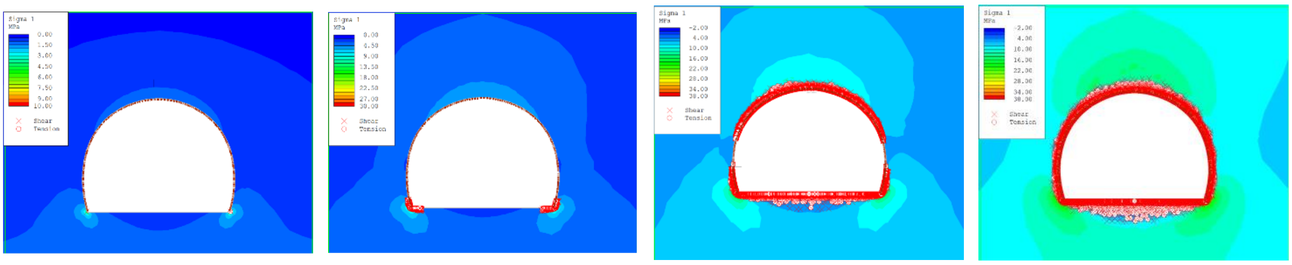
Depth=1*D* Depth =4*D* Depth =7*D* Depth =8*D* Depth =12*D* Depth =15*D*

Figure 5: Stress Contours at different Tunnel Depths (Close-up Views)

4.2 Plastic Zone Development around Excavation Profile

The concept of loosened rock after excavation in empirical approaches can be correlated to the plastic zone in numerical modelling. Therefore, the influence of tunnel depth and rock mass quality to the plastic zone development around excavation profile were studied as shown in Figure 6 and Figure 7 below.

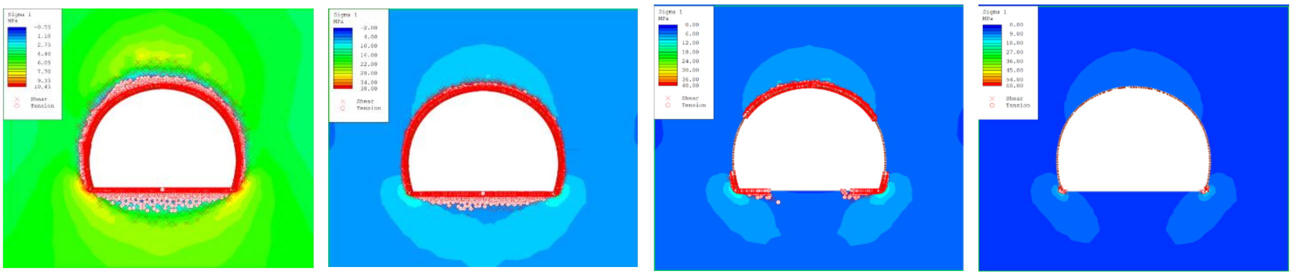
It was noted that the extent of plastic zone increased with overburden depth. At shallow depth, plastic zones were only observed at corners. With the increasing depth, the plastic zone extended to the tunnel crown, invert and further to the wall around the entire excavation profile. The thickness of plastic zone was approximately 1 to 2.5 m.



Depth=1*D* Depth=5*D* Depth=9*D* Depth=15*D*

Figure 6: Plastic Zone around Excavation Profile at different Tunnel Depths

It was also observed that the plastic zone decreased significantly with better rock mass quality. For very poor rock (Q=0.1), the plastic zone developed all around the profile with thickness of about 1 to 2.5 m. For Q=1, the plastic zone started to decrease. For Q=4, the plastic zone only developed at crown and corners. For very good rock (Q=40), only small area of plastic zones at corners were observed.



Q=0.1 (Very poor) Q=1 Q=4 Q=40 (Very good)

Figure 7: Plastic Zone around Excavation Profile for different Rock Mass Quality Q

4.3 Comparison of Estimated Rock Support Pressure

The comparison of estimated rock support pressure at tunnel crown from the empirical approaches and numerical modelling are summarized in Figure 8 to Figure 11.

For most of the cases, the support pressure at tunnel crown from numerical modelling was much smaller than that from empirical approaches. It was found that the estimation by Grimstad & Barton (1993) was the closer one to the numerical results than Terzaghi (1946) except for poor rock mass conditions.

In general, it was observed that the influence of in-situ stress ratio and rock mass quality were significant to the support pressure. Also, the support pressure increased slowly with increasing tunnel depth. However, the corresponding influence of uniaxial compressive strength, intact rock modulus, poisson's ratio and tunnel width were found to be negligible for tunnel lining design in hard rock.

After excavation, the in-situ stresses are redistributed around the tunnel profile. However, at moderate depth (around 100m), these induced stresses are very small when compare to the compressive strength of strong rock (the baseline UCS is 75 MPa). Same finding in Figure 8 (left) was observed where the uniaxial compressive strength was found to be insignificant to the support pressure for tunnel lining design in hard rock.

Also, Figure 8 (right) and Figure 9 (left) illustrate that the influence of deformation parameters including the intact rock modulus and poisson's ratio to the pressure were insignificant for tunnel lining design in hard rock.

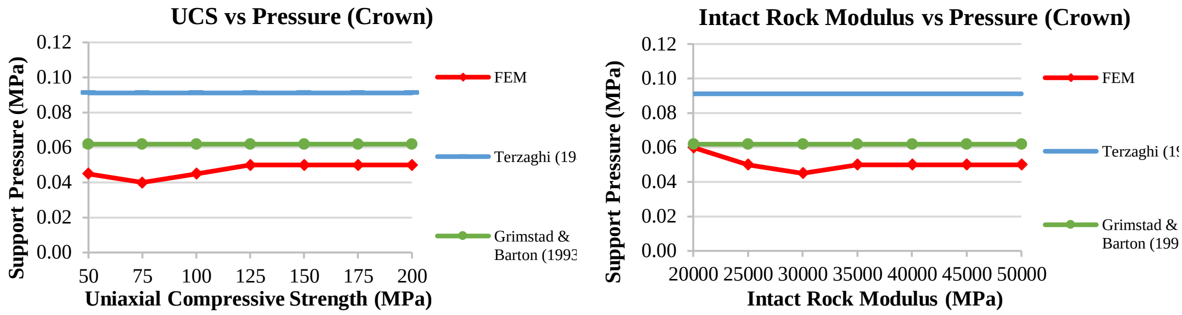


Figure 8: Influence of UCS (left) and Intact Rock Modulus (right) to Rock Support Pressure (Crown)

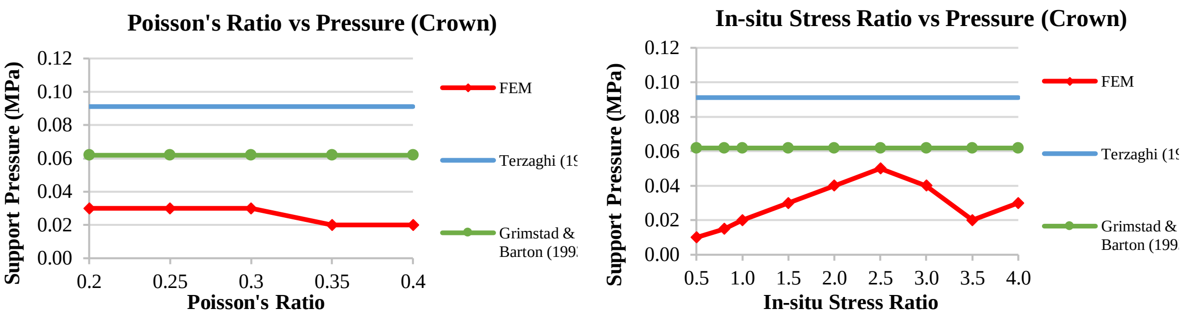


Figure 9: Influence of Poisson's Ratio (left) and
In-situ Stress Ratio (right) to Rock Support Pressure (Crown)

According to GEO (2018), the in-situ stress ratio in Hong Kong at shallow depth ranges from 1.4 to 2.5 and drops to about 1 to 1.5 at a depth below 100 m. Within this common range of 1 to 2.5, the results from numerical results in Figure 9 (right) revealed that the rock support pressure at crown increased with the in-situ stress ratio.

Figure 10 below indicates an exponential decay function for the relationship of rock mass quality with support pressure at crown. For very poor rock, the estimated pressure by numerical modelling was larger than that by empirical approaches. This could probably be explained by the wrong application of same ground relaxation and lining stiffness for different ground conditions, which led to excessive ground deformation and yielding failure. This could be avoided if a stronger lining was assigned to support the ground. A detailed study could be carried out for further investigations.

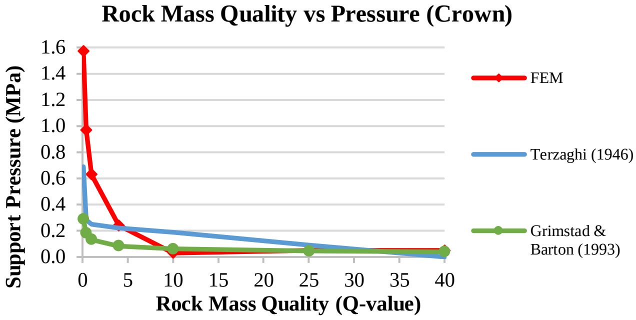


Figure 10: Influence of Rock Mass Quality to Rock Support Pressure (Crown)

A slow increasing trend of support pressure with tunnel depth has been discussed in Section 4.1. The same trend was observed from FEM results in Figure 11 (left). Nevertheless, the estimated pressure at tunnel depth of about 200 m was still smaller than all empirical approaches. The estimation by Grimstad & Barton (1993) is still conservative to be adopted for tunneling at moderate depth.

The influence of excavation width to support pressure has been a controversial topic in rock tunnelling. Some previous studies have confirmed the dependency when a flat tunnel roof was used or at very poor rock conditions. Figure 11 (right) presents that the influence of tunnel width for a horseshoe profile to support pressure at crown was negligible for tunnel lining design in hard rock.

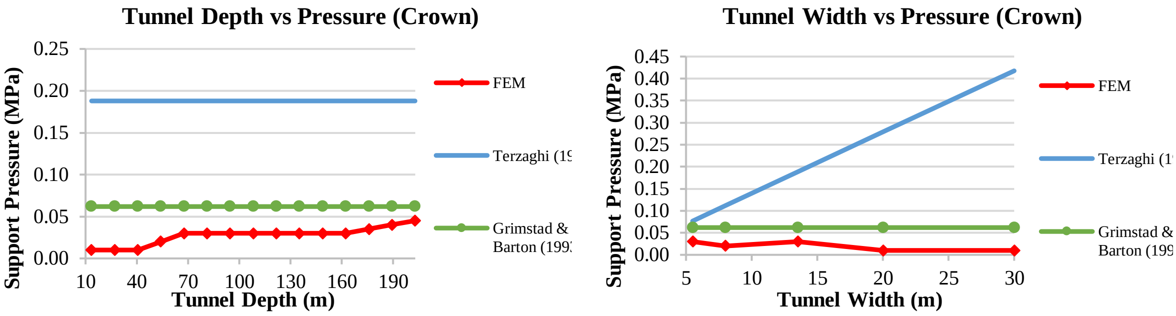


Figure 11: Influence of Tunnel Depth (left) and
Tunnel Width (right) to Rock Support Pressure (Crown)

The detailed comparison for support pressure at walls are not discussed in this paper.

5 Conclusions and Recommendations

Empirical approaches are conservative to estimate the rock support pressure acting on tunnel lining and therefore is very useful during the planning and preliminary design phases. However, it could be too conservative on certain occasions which cannot provide a cost-effective engineering solution. When more ground investigation data is available during the detailed design phase in later stage, the use of numerical modelling shall be explored for design optimisation in order to save construction cost and time. Particular attention should be given when planning the ground investigation proposal for tunnel projects. For example, laboratory tests for uniaxial compressive strength, intact rock modulus & poisson's ratio are frequently ordered by geotechnical engineers, but they are not particularly useful to the design of permanent tunnel lining in hard rock, which dominant the solid geology in Hong Kong.

Alternatively, other types of ground investigation such as the use of horizontal directional coring (HDC) should be explored to determine the representative rock mass conditions and details of inferred faults/weakness zones along the proposed tunnel alignment in order to identify the locations of poor rock. Furthermore, although this paper states that larger-span excavation would experience the same support pressure, lining with larger-span will still sustain a much higher bending moment during structural design. Therefore, field tests for in-situ stress ratio measurement (such as hydraulic fracturing or over-coring) are recommended for projects where large-span tunnels and caverns are proposed.

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