

Chapter 10:

Bayesian Approximation Techniques for Gompertz Distribution

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DOI: <https://doi.org/10.21467/books.44.10>

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Introduction:

The Gompertz distribution plays an important role in modeling survival times, human mortality and actuarial tables. Gompertz probability distribution has many useful applications in areas of technology, medical, biological, and natural sciences. The Gompertz distribution was introduced by Gompertz (1825), and many authors have contributed to the statistical methodology and characterization of this distribution. Ismail (2010) discussed Bayes estimation for unknown parameters of Gompertz distribution and acceleration factors under partially accelerated life tests with Type-I censoring. Based on progressive first-failure censoring plans. Soliman et al. (2012) studied Bayes and frequentist estimators for two-parameter Gompertz distribution. Feroze and Aslam (2013) obtained point and interval estimates for the parameters of the two-component mixture of the Gompertz model based on Bayes Method along with posterior predictions for the future value from model. Sarabia et al. (2014) explored several properties of the Gompertz distribution when lifetime or other kinds of data available fully observed. Prakash (2016) discussed about the Bayes prediction bound length under different censoring plans and statistical inference based on a random scheme under progressive Type-II censored data for Gompertz model. Reyad et al. (2016) introduced a comparative study for the E-Bayesian criteria with three various Bayesian approaches; Bayesian, hierarchical Bayesian and empirical Bayesian.

The probability density function of Gompertz distribution is given by

$$f(x) = \delta e^x e^{-\delta(e^x-1)} \quad ; \quad x > 0; \delta > 0 \quad (10.1)$$



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The likelihood function for (10.1) is given by

$$L(x) = \delta^n \prod_{i=1}^n e^x e^{-\delta(e^x-1)}$$

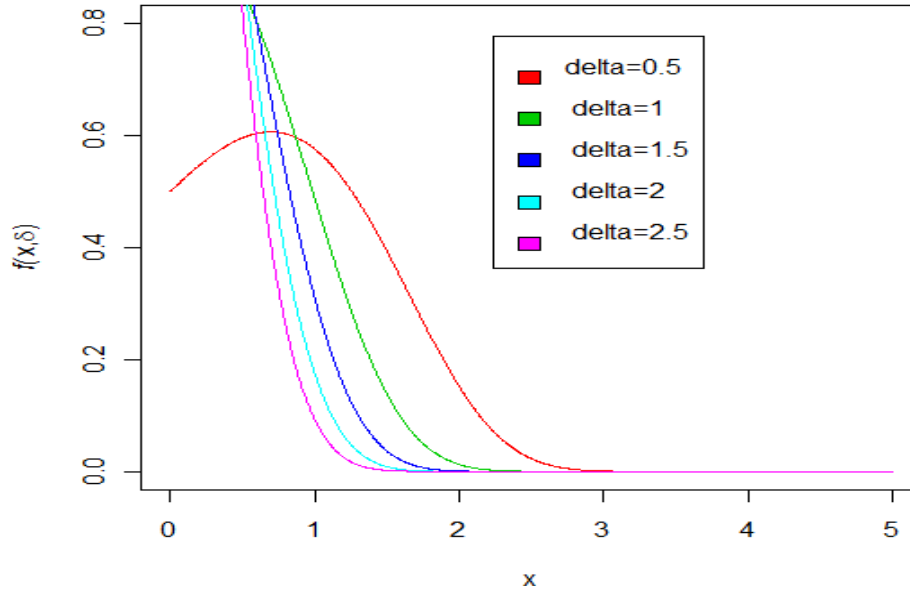


Figure 10.1: represents probability density function of Gompertz distribution under different values of parameters.

The Bayesian analysis is theoretically simple and probabilistically elegant. When posterior distribution is expressible in terms of complex analytical function and requires thorough calculation because of its numerical implementations, an approximate and large sample behavior of posterior distribution is studied. This is significant for two reasons: (a) asymptotic results provide valuable first order approximations when actual samples are relatively large, and (b) objective Bayesian methods obviously depend on the asymptotic properties of the assumed model. Thus, our current reading focuses to obtain the estimates of shape parameter of Gompertz distribution using two Bayesian approximation techniques i.e. normal approximation, T-K approximation.

Bayes Estimate of Shape Parameter of Gompertz Distribution using Normal Approximation:

If the posterior distribution $\Psi(\delta | x)$ is unimodal and roughly symmetric, it is convenient to approximate it by a normal distribution centered at the mode, yielding the approximation

$$\Psi(\delta | x) \sim \hat{N}\left(\hat{\delta}, [I(\hat{\delta})]^{-1}\right)$$

$$\text{where } I(\hat{\delta}) = -\frac{\partial^2 \log P(\delta | y)}{\partial \delta^2} \quad (10.2)$$

If the mode, $\hat{\delta}$ is in the interior parameter space, then $I(\delta)$ is positive; if $\hat{\delta}$ is a vector parameter, then $I(\delta)$ is a matrix. Some good sources on the topic is provided by Sultan et al. (2015).

In our study the normal approximations of Gompertz distribution under different priors is obtained as under:

Under extension of Jeffrey's prior $\varphi(\delta) \propto \left(\frac{1}{\delta}\right)^m$; $m \in R^+$, the posterior distribution for δ is as

$$\Psi(\delta | x) \propto \delta^{n-m} e^{\sum_{i=1}^n x_i} e^{-\delta \sum_{i=1}^n (e^{x_i} - 1)} \quad (10.3)$$

from which the posterior mode is obtained as $\hat{\delta} = \frac{n-m}{\sum_{i=1}^n (e^{x_i} - 1)}$

$$\text{and } [I(\hat{\delta})]^{-1} = \frac{n-m}{\left[\sum_{i=1}^n (e^{x_i} - 1)\right]^2}$$

Thus, the posterior distribution can be approximated as

$$\Psi(\delta | x) \sim N\left(\frac{n-m}{\sum_{i=1}^n (e^{x_i} - 1)}; \frac{n-m}{\left[\sum_{i=1}^n (e^{x_i} - 1)\right]^2}\right)$$

Under the Inverse Levy prior $\varphi(\delta) \propto \delta^{-1/2} e^{-\frac{\delta c}{2}}$; $c > 0; \delta > 0$, where c is the known hyper parameter, the posterior distribution for δ is as

$$\Psi(\delta | x) \propto \delta^{n-1/2} e^{\sum_{i=1}^n x_i} e^{-\delta \left[c/2 + \sum_{i=1}^n (e^{x_i} - 1)\right]}$$

from which the posterior mode is obtained as $\hat{\delta} = \frac{n-1/2}{c/2 + \sum_{i=1}^n (e^{x_i} - 1)}$

$$\text{and } [I(\hat{\delta})]^{-1} = \frac{n-1/2}{\left[c/2 + \sum_{i=1}^n (e^{x_i} - 1)\right]^2}$$

Thus, the posterior distribution can be approximated as

$$\Psi(\delta | x) \sim N \left(\frac{n-1/2}{c/2 + \sum_{i=1}^n (e^{x_i} - 1)}; \frac{n-1/2}{\left[c/2 + \sum_{i=1}^n (e^{x_i} - 1) \right]^2} \right)$$

Under gamma prior $\varphi(\delta) \propto \delta^{a-1} e^{-b\delta}$; $a, b > 0; \delta > 0$ where a, b are the known hyper parameters, the posterior distribution for δ is approximated as

$$\Psi(\delta | x) \sim N \left(\frac{n+a-1}{\sum_{i=1}^n (e^{x_i} - 1) + b}; \frac{n+a-1}{\left[\sum_{i=1}^n (e^{x_i} - 1) + b \right]^2} \right)$$

Bayes Estimate of Shape Parameter of Gompertz Distribution using T-K (Laplace) Approximation:

Tierney and Kadane (1986) gave Laplace method to evaluate $E(h(\delta) | x)$ as

$$E(h(\delta) | x) \cong \frac{\hat{\omega}^*}{\bar{\omega}} \exp \left\{ n h(\hat{\delta}^*) - n h(\hat{\delta}) \right\} \quad (10.4)$$

where $n h''(\hat{\delta}) = \ln \Psi(\delta | x)$; $n h''^*(\hat{\delta}^*) = \ln \Psi(\theta | x) + \ln h(\delta)$;

$$\hat{\omega}^2 = -[n h''(\hat{\delta})]^{-1}; \hat{\omega}^{*2} = -[n h''^*(\hat{\delta}^*)]^{-1}$$

Under extension of Jeffrey's prior $\varphi(\delta) \propto \left(\frac{1}{\delta}\right)^m$; $m \in \mathbb{R}^+$, $m \in \mathbb{R}^+$ the posterior distribution for δ is given in (10.3)

$$n h(\delta) = (n-m) \ln \delta - \delta \sum_{i=1}^n (e^{x_i} - 1)$$

which implies $\hat{\delta} = \frac{n-m}{\sum_{i=1}^n (e^{x_i} - 1)}$ that maximizes $n h(\lambda)$ since $n h''(\delta) = -\frac{n-m}{\delta^2} < 0$

Similarly $n h^*(\delta^*) = n h(\delta) + \ln h(\delta) = (n-m+1) \ln \delta - \delta \sum_{i=1}^n (e^{x_i} - 1)$

From which $\hat{\delta}^* = \frac{n-m+1}{\sum_{i=1}^n (e^{x_i} - 1)}$ that maximizes $nh^*(\delta^*)$ since

$$nh^{*''}(\delta^*) = -\frac{n-m+1}{\delta^2} < 0$$

Thus the maximum of $nh(\delta)$ and $nh^*(\delta^*)$ are given by

$$nh(\hat{\delta}) = \ln \left(\frac{n-m}{\sum_{i=1}^n (e^{x_i} - 1)} \right)^{n-m} - (n-m) \quad \&$$

$$nh^*(\hat{\delta}^*) = \ln \left(\frac{n-m+1}{\sum_{i=1}^n (e^{x_i} - 1)} \right)^{n-m+1} - (n-m+1)$$

respectively.

The estimates of variances are given by

$$\varpi = \frac{\partial^2 nh(\delta)}{\partial \delta^2} \Big|_{\delta=\hat{\delta}} = \frac{(n-m)^{1/2}}{\sum_{i=1}^n (e^{x_i} - 1)}$$

$$\& \quad \varpi^* = \frac{\partial^2 nh(\delta^*)}{\partial \delta^{*2}} \Big|_{\delta^*=\hat{\delta}^*} = \frac{(n-m+1)^{1/2}}{\sum_{i=1}^n (e^{x_i} - 1)}$$

So we have $E(\delta | x) \cong \frac{\varpi^*}{\varpi} \exp \{ nh(\hat{\delta}^*) - nh(\hat{\delta}) \}$

$$= \left(\frac{n-m+1}{\sum_{i=1}^n (e^{x_i} - 1)} \right) \left(\frac{n-m+1}{n-m} \right)^{n-m+1/2} e^{-1}$$

Note that the relative error to exact the posterior mean $\frac{n-m+1}{\sum_{i=1}^n (e^{x_i} - 1)}$ is

$$\left(\frac{n-m+1}{n-m} \right)^{n-m+1/2} e^{-1}.$$

To determine the second moment, assume $h(\delta) = \delta^2$

$$\therefore n h^*(\delta^*) = (n - m + 2) \ln \delta - \delta \sum_{i=1}^n (e^{x_i} - 1)$$

$$\text{From which } E(\delta^2 | x) = \left(\frac{n - m + 2}{\sum_{i=1}^n (e^{x_i} - 1)} \right)^2 \left(\frac{n - m + 2}{n - m} \right)^{n-m+1/2} e^{-2}$$

Thus, variance is given by

$$\left[\left(\frac{n - m + 2}{\sum_{i=1}^n (e^{x_i} - 1)} \right)^2 \left(\frac{n - m + 2}{n - m} \right)^{n-m+1/2} e^{-2} - \left[\left(\frac{n - m + 1}{\sum_{i=1}^n (e^{x_i} - 1)} \right) \left(\frac{n - m + 1}{n - m} \right)^{n-m+1/2} e^{-1} \right]^2 \right]$$

Under Gamma prior $\varphi(\delta) \propto \delta^{a-1} e^{-b\delta}$; $a, b > 0; \delta > 0$

$$E(\delta | x) = \left(\frac{n + a - 1}{b + \sum_{i=1}^n (e^{x_i} - 1)} \right) \left(\frac{n + a}{n + a - 1} \right)^{n+a+1/2} e^{-1}$$

where the relative error exact to the posterior mean $\left(\frac{n + a - 1}{b + \sum_{i=1}^n (e^{x_i} - 1)} \right)$ is

$$\left(\frac{n + a}{n + a - 1} \right)^{n+a+1/2} e^{-1}.$$

$$\text{Further } E(\delta^2 | x) = \left(\frac{(n + a + 1)(n + a - 1)}{\left[b + \sum_{i=1}^n (e^{x_i} - 1) \right]^2} \right) \left(\frac{n + a}{n + a - 1} \right)^{n+a+1/2} e^{-2}$$

Thus, variance is given by

$$\left[\left(\frac{(n + a + 1)(n + a - 1)}{\left[b + \sum_{i=1}^n (e^{x_i} - 1) \right]^2} \right) \left(\frac{n + a}{n + a - 1} \right)^{n+a+1/2} e^{-2} - \left[\left(\frac{n + a - 1}{b + \sum_{i=1}^n (e^{x_i} - 1)} \right) \left(\frac{n + a}{n + a - 1} \right)^{n+a+1/2} e^{-1} \right]^2 \right]$$

Under inverse levy prior $\varphi(\delta) \propto \delta^{-1/2} e^{-\frac{\delta c}{2}}; c > 0; \delta > 0$

$$E(\delta | x) = \left(\frac{n+1/2}{c/2 + \sum_{i=1}^n (e^{x_i} - 1)} \right) \left(\frac{n+1/2}{n-1/2} \right)^n e^{-1}$$

where the relative error exact to the posterior mean $\left(\frac{n+1/2}{c/2 + \sum_{i=1}^n (e^{x_i} - 1)} \right)$ is $\left(\frac{n+1/2}{n-1/2} \right)^n e^{-1}$

$$\text{Further } E(\delta^2 | x) = \left(\frac{n+3/2}{c/2 + \sum_{i=1}^n (e^{x_i} - 1)} \right)^2 \left(\frac{n+3/2}{n-1/2} \right)^n e^{-2}$$

Thus, variance is given by

$$\left(\frac{n+3/2}{c/2 + \sum_{i=1}^n (e^{x_i} - 1)} \right)^2 \left(\frac{n+3/2}{n-1/2} \right)^n e^{-2} - \left[\left(\frac{n+1/2}{c/2 + \sum_{i=1}^n (e^{x_i} - 1)} \right) \left(\frac{n+1/2}{n-1/2} \right)^n e^{-1} \right]^2$$

Real life example 10.1

To examine the applicability of the results, real life data sets are analyzed. The data represents the survival times of 121 patients with breast cancer obtained from a large hospital which is widely reported in some literatures like Ramos et al. (2013)).

0.3,0.3,4.0,5.0,5.6,6.2,6.3,6.6,6.8,7.4,7.5,8.4,8.4,10.3,11.0,11.8,12.2,12.3,13.5,14.4,14.4,14.8,15.5, 15.7,16.2,16.3,16.5,16.8,17.2,17.3,17.5,17.9,19.8,20.4,20.9,21.0,21.0,21.1,23.0,23.4,23.6,24.0,24. 0,27.9,28.2,29.1,30.0,31.0,31.0,32.0,35.0,35.0,37.0,37.0,37.0,38.0,38.0,38.0,39.0,39.0,40.0,40.0,4 0.0,41.0,41.0,41.0,42.0,43.0,43.0,43.0,44.0,45.0,45.0,46.0,46.0,47.0,48.0,49.0,51.0,51.0,51.0,52.0, 54.0,55.0,56.0,57.0,58.0,59.0,60.0,60.0,60.0,61.0,62.0,65.0,65.0,67.0,67.0,68.0,69.0,78.0,80.0,83. 0,88.0,89.0,90.0,93.0,96.0,103.0,105.0,109.0,109.0,111.0,115.0,117.0,125.0,126.0,127.0,129.0,12 9.0,139.0,154.0.

Real life example 10.2

Consider the data of survival times of 45 gastric cancer patients given chemotherapy and radiation treatment (Bekker et al. 2000).

0.047, 0.115, 0.121, 0.132, 0.164, 0.197, 0.203, 0.260, 0.282, 0.296, 0.334, 0.395, 0.458, 0.466, 0.501, 0.507, 0.529, 0.534, 0.540, 0.570, 0.641, 0.644, 0.696, 0.841, 0.863, 1.099, 1.219, 1.271, 1.326, 1.447, 1.485, 1.553, 1.581, 1.586, 2.178, 2.343, 2.416, 2.444, 2.825, 2.830, 3.578, 3.658, 3.743, 3.978, 4.033

The Bayes estimates and posterior standard error (given in parenthesis) for both the examples under normal and T-K (Laplace) approximation based on non- informative and informative priors have been presented in table 10.1, 10.2

Table 10.1: Posterior estimates and posterior standard error (in parenthesis) under normal and T-K (Laplace) approximations

	Jeffrey's prior			Gamma prior			Inverse levy prior		
	m=0.5	m=1	m=1.5	a=b=1	a=b=2	a=b=3	c=1	c=2	c=3
$\hat{\delta}_{NA}$	0.0925 (0.0172)	0.0909 (0.0171)	0.0893 (0.0167)	0.0937 (0.0171)	0.0966 (0.0173)	0.0994 (0.0175)	0.09221 (0.0170)	0.0919 (0.0167)	0.0916 (0.0166)
$\hat{\delta}_{LA}$	0.0956 (0.0173)	0.0941 (0.0172)	0.0925 (0.0170)	0.0969 (0.0185)	0.0997 (0.0182)	0.1025 (0.0180)	0.0954 (0.0171)	0.0953 (0.0169)	0.0951 (0.0168)

Table 10.2: Posterior estimates and posterior standard error (in parenthesis) under normal and T-K (Laplace) approximations:

	Jeffrey's prior			Gamma prior			Inverse levy prior		
	m=0.5	m=1	m=1.5	a=b=1	a=b=2	a=b=3	c=1	c=2	c=3
$\hat{\delta}_{NA}$	0.1336 (0.0309)	0.1321 (0.0299)	0.1306 (0.0198)	0.1347 (0.02008)	0.1372 (0.0202)	0.1398 (0.0204)	0.1332 (0.0200)	0.1328 (0.0199)	0.1324 (0.0188)
$\hat{\delta}_{LA}$	0.1366 (0.02025)	0.1351 (0.02014)	0.1336 (0.02002)	0.1377 (0.02031)	0.1402 (0.02046)	0.1428 (0.02061)	0.1364 (0.02022)	0.1362 (0.02019)	0.1360 (0.02016)

Simulation study

In our simulation study we have generated a sample of sizes $n=20, 50, 75$ to see the result of small, medium, and large samples on the estimators. The results are simulated 5000 times and the average of the results has been presented in the tables 10.3, 10.4. To inspect the performance of Bayesian estimates for shape parameter of Gompertz distribution under different approximation techniques, estimates are obtainable along with posterior standard error given in parenthesis in the below tables.

Table10.3: Posterior estimates and posterior standard deviation (in parenthesis) under normal approximation

n	δ	Jeffrey's prior			Gamma prior			Inverse levy prior		
		m=0.5	m=1.0	m=1.5	a=b=1	a=b=2	a=b=3	c=1	c=2	c=3
20	0.9	0.9271 (0.2099)	0.9032 (0.2072)	0.8795 (0.2044)	0.9076 (0.2029)	0.9116 (0.1989)	0.9153 (0.1951)	0.8849 (0.2004)	0.8465 (0.1917)	0.8113 (0.1837)
	1.5	1.3021 (0.2948)	1.2687 (0.2910)	1.2353 (0.2872)	1.2518 (0.2799)	1.2370 (0.2699)	1.2238 (0.2609)	1.2205 (0.2764)	1.1486 (0.2601)	1.0847 (0.2456)
	2.5	1.8949 (0.4291)	1.8463 (0.4235)	1.7977 (0.4179)	1.7713 (0.3961)	1.7086 (0.3728)	1.6553 (0.3529)	1.7271 (0.3911)	1.5865 (0.3592)	1.4672 (0.3322)
50	0.9	1.1378 (0.1617)	1.1263 (0.1609)	1.1148 (0.1601)	1.1234 (0.1588)	1.1207 (0.1569)	1.1181 (0.1551)	1.1122 (0.1581)	1.0878 (0.1546)	1.0644 (0.1512)
	1.5	1.1956 (0.1699)	1.1835 (0.1690)	1.1541 (0.1682)	1.1792 (0.1667)	1.1751 (0.1645)	1.1711 (0.1624)	1.1674 (0.1659)	1.1405 (0.1621)	1.1148 (0.1584)
	2.5	1.5784 (0.2243)	1.5625 (0.2232)	1.5466 (0.2221)	1.5451 (0.2185)	1.5288 (0.2141)	1.5134 (0.2098)	1.5297 (0.2174)	1.4838 (0.2109)	1.4406 (0.2047)
75	0.9	0.8118 (0.0941)	0.8009 (0.0934)	0.8009 (0.0934)	0.8084 (0.0933)	0.8104 (0.0929)	0.8124 (0.0925)	0.8030 (0.0930)	0.7944 (0.0920)	0.7861 (0.0911)
	1.5	1.1887 (0.1377)	1.1807 (0.1372)	1.1727 (0.1367)	1.1779 (0.1360)	1.1751 (0.1347)	1.1724 (0.1336)	1.1701 (0.1355)	1.1519 (0.1334)	1.1344 (0.1314)
	2.5	1.5473 (0.1792)	1.5369 (0.1786)	1.5265 (0.1781)	1.5259 (0.1762)	1.5155 (0.1738)	1.5054 (0.1715)	1.5158 (0.1756)	1.4855 (0.1721)	1.4565 (0.1687)

Table10.4: Posterior estimates and posterior standard error (in parenthesis) under T-K approximation

n	δ	Jeffrey's prior			Gamma prior			Inverse levy prior		
		m=0.5	m=1	m=1.5	a=b=1	a=b=2	a=b=3	c=1	c=2	c=3
20	0.9	0.9748 (0.2152)	0.9510 (0.2126)	0.9272 (0.2099)	0.9532 (0.2079)	0.9552 (0.2036)	0.9571 (0.1995)	0.9521 (0.2102)	0.9305 (0.2054)	0.9099 (0.2009)
	1.5	1.3691 (0.3023)	1.3357 (0.2986)	1.3024 (0.2948)	1.3147 (0.2868)	1.2961 (0.2763)	1.2797 (0.2667)	1.3249 (0.2925)	1.2834 (0.2834)	1.2445 (0.2748)
	2.5	1.9925 (0.4399)	1.9439 (0.4345)	1.8953 (0.4291)	1.8603 (0.4058)	1.7903 (0.3816)	1.7308 (0.3608)	1.9002 (0.4195)	1.8160 (0.4010)	1.7390 (0.3840)
50	0.9	1.1608 (0.1633)	1.1493 (0.1625)	1.1378 (0.1617)	1.1427 (0.1786)	1.1427 (0.1584)	1.1397 (0.1565)	1.1476 (0.1614)	1.1347 (0.1596)	1.1221 (0.1579)
	1.5	1.2198 (0.1716)	1.2077 (0.1707)	1.1956 (0.1699)	1.2028 (0.1684)	1.1981 (0.1661)	1.1937 (0.1639)	1.2052 (0.1696)	1.1911 (0.1676)	1.1771 (0.1656)
	2.5	1.6104 (0.2266)	1.5944 (0.2254)	1.5785 (0.2243)	1.5761 (0.2206)	1.5588 (0.2161)	1.5425 (0.2118)	1.5851 (0.2230)	1.5606 (0.2196)	1.5369 (0.2162)
75	0.9	0.8227 (0.0946)	0.8172 (0.0943)	0.8118 (0.0941)	0.8192 (0.0939)	0.8211 (0.0935)	0.8230 (0.0931)	0.8182 (0.0941)	0.8138 (0.0936)	0.8094 (0.0931)
	1.5	1.2046 (0.1386)	1.1967 (0.1381)	1.1887 (0.1377)	1.1936 (0.1369)	1.1906 (0.1356)	1.1877 (0.1344)	1.1951 (0.1375)	1.1857 (0.1364)	1.1765 (0.1354)
	2.5	1.5680 (0.1804)	1.5577 (0.1798)	1.5473 (0.1792)	1.5463 (0.1773)	1.5354 (0.1749)	1.5250 (0.1726)	1.5519 (0.1786)	1.5361 (0.1767)	1.5207 (0.1750)

Conclusion

We presented approximate to Bayesian integrals of Gompertz distribution depending upon numerical integration and simulation study and showed how to study posterior distribution by means of simulation study. From the findings of above tables (1, 2, 3, 4) it has been found that the large sample distribution could be improved when prior is taken into account. In all cases (simulated data as well as real life data) normal approximation, T-K approximation, Bayesian estimates under informative priors are better than those under non-informative priors especially the Inverse levy distribution proves to be efficient with minimum posterior standard deviation. Further we accomplish that the posterior standard deviation based on different priors tends to decrease with the increase in sample size. It indicates that the estimators attained are consistent. It can also be detected that the performance of Bayes estimates under informative priors (inverse levy) is better than non-informative prior.

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How to Cite this Chapter:

Sultan, Humaira. "Bayesian Approximation Techniques for Gompertz Distribution." *Bayesian Analysis and Reliability Estimation of Generalized Probability Distributions*, edited by Afaq Ahmad, AIJR Publisher, 2019, pp. 108–119, ISBN: 978-81-936820-7-4, DOI: [10.21467/books.44.10](https://doi.org/10.21467/books.44.10).

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