Chapter 7:

Designing Accelerated Life Testing for Product Reliability Under Warranty Prospective

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Additional information is available at the end of the chapter

Introduction

In the current era, every manufacturer producing goods want to increase their sales by providing some incentives to customers in the form of warranty and guaranty. It is a written statement presented by Manufacturers to the customers, promising them to repair or replace the product they purchased, if necessary, within a specified period of time. It is additionally a way of advertising the standard of the merchandise and thereby boosting sales. A detailed review of assorted problems associated with product warranties will be found in Blischke and Murthy (1992a, 1992b, 1994) and Chien (2010).

One of the types of warranty policies is rebate warranty. Under this scheme, a customer (buyer) is refunded by some percent of money (sales price) if the product meets the defect during the warranty time spam. Batteries and tires are the items sold under this warranty scheme. Common forms include: lump sun, and pro-rata rebates. Other issues and discussions related to warranty policies can be found in Mitra and Patankar (1993), Murthy (1990), Murthy and Blischke (1992).

The manufacturers shall only provide these incentives when they have the faith on the products that their product has the ability to serve at least for the time period warranty is given. Therefore, it is essential for manufacturers to test the reliability and performance of the products before letting them serve in the market. This can be done by using accelerated life testing (ALT) on products, where products are put at higher stresses than normal to induce failure and predict their life under normal use conditions. ALT also helps Manufacturers to predict the various costs associated with the product under warranty policy. The main aim of ALT is to find the failure data of such products and systems by subjecting them to the higher levels of stresses. Hence accelerated life testing is needed to quickly provide the information about the life distribution of products.

For analysing ALT efficiently and to obtain performance data, the experimenter needs to determine the testing method, statistical model, form of the life data and a suitable statistical method. Analysing these measures properly, provides the best estimates of the product's life and performance under usual conditions.

There are researchers who combined accelerating life testing and warranty models. GuangbinYang, (2010), provided a method for describing the warranty cost, and its confidence interval. El-Dessoukey (2015) used accelerated life tests along with Exponentiated Pareto distribution to describe age replacement policy under warranty policy.

The article describes how to use accelerated life testing procedures for predicting the cost of age replacement of units or products under warranty policy. Under constant stress, the generalized exponential distribution is assumed to cover the lifetimes of the products. The chapter also describes the age replacement policy in the combination of pro-rata rebate warranty for non-repairable units.

Model Description and Test Method

ALT is a best used method for reliability and life prediction of systems or components. Designing the ALT plan, needs to determine the following:

- **i.** The statistical distribution of failure times of products.
- **ii.** Type of data used, complete or censored.
- **iii.** Type of censoring scheme.
- **iv.** The type of stress to be used.
- **v.** The stress level selected.
- **vi.** The percentage of test units allocated for each stress level.
- **vii.** The mathematical model describing the relation between life and stress (life-stress relationship).

This study is dealt with constant stress and type-I censored data under the assumption that the lifetimes of the units follows generalized exponential distribution. There are k levels of high stresses $V_j, j = 1, 2, ..., K$ and assume that V_u is the normal use condition satisfying $V_u < V_1 < V_2 < .... < V_k$. At each stress level, there are n_j units put on test and the experiment terminates once the number of failures r_j among these n_j units are observed.

For the detailed review of constant stress ALT one may refer to Abdel-Ghaly et al. (1998), El-Dessouky (2001), Attia et al. (2011), and Attia et al. (2013). The studies have shown that the two-parameter Generalised Exponential distribution can selectively used in place of two-parameter gamma and two-parameter Weibull distributions for analysing many lifetime data (see, Gupta and Kundu 1999). The probability density function (pdf) of a generalised Exponential distribution is given by

$$f(t_{ij},\alpha_j,\beta)=\frac{\alpha_j}{\beta}e^{-\frac{t_{ij}}{\beta}}\left(1-e^{-\frac{t_{ij}}{\beta}}\right)^{\alpha_j-1} \;;\; \alpha_j>0,\beta>0,t_{ij}>0 \tag{7.1}$$

where, the shape papameter $\alpha_j > 0,$ and the scale parameter $\beta > 0,$ hence denoted by $GE(\alpha,\beta)$. The CDF of generalised Exponential distribution is

$$F(t_{ij},\alpha_j,\lambda)=\left(1-e^{-\frac{t_{ij}}{\beta}}\right)^{\alpha_j} \;;\; \alpha_j>0,\beta>0,t_{ij}>0 \tag{7.2}$$

The survival function is given by

$$S(t_{ij},\alpha_j,\lambda)=1-\left(1-e^{-\frac{t_{ij}}{\beta}}\right)^{\alpha_j} \tag{7.3}$$

The failure rate or hazard rate is given by

$$h(t_{ij},\alpha_j,\lambda)=\frac{\frac{\alpha_j}{\beta}e^{-\frac{t_{ij}}{\beta}}\left(1-e^{-\frac{t_{ij}}{\beta}}\right)^{\alpha_j-1}}{1-\left(1-e^{-\frac{t_{ij}}{\beta}}\right)^{\alpha_j}}$$

The studies have shown that GE distribution very well represents the life of modern products. The hazard function behaves like as of gamma distribution, see Ahmad (2010).

It is also assumed that the stress $V_j,\ j=1,2,...,k$ only affects the shape parameter of the Generalised exponential model, α_j through a life stress model called power rule model given by:

$$\alpha_j=CV_j^{-p};\;\; C>0\,,\, p>0\,,\;\; j=1,2,...,k \tag{7.4}$$

where C is the proportionality constant, and p is the power of the applied stress, are the two model parameters.

The Estimation Procedure

The likelihood function of an observation t (failure time of an item) is developed at stress level V_j. Since at each stress level V_j, there are n_j units put under test. Here, the total population experimental units are N= $\sum_{i=1}^{k} n_j$. Applying type-I censoring at each stress level, it can be seen that the once the censoring time "t_0" is reached the experiment automatically terminates. Assume that $r_j(\leq n_j)$ failures are observed at the jth stress level prior to the termination of the test and $(n_j - r_j)$ units still survived. Therefore, likelihood function becomes:

$$L(\alpha_j, C, \beta) = \prod_{j=1}^{k} \frac{n_j}{(n_j - r_j)!} \left[\prod_{i=1}^{r_j} f(t_{ij}; \alpha_j, C, \beta) \right] \left[1 - F(t_0) \right]^{n_j - r_j} \qquad (7.5)$$

Where t_0 is the time of cessation of the test.

Using InL to denote the natural logarithm of $L(\alpha_j, C, \beta)$, then we have

$$InL = K + \sum_{j=1}^{k} r_j (InC - In\beta) - p \sum_{j=1}^{k} r_j InV_j - \sum_{j=1}^{k} \sum_{i=1}^{r_j} \frac{t_{ij}}{\beta} + \sum_{j=1}^{k} \sum_{i=1}^{r_j} (CV_j^{-p} - 1) In(W(t_{ij})) + \sum_{j=1}^{k} (n_j - r_j) In\left[1 - (W(t_0))^{CV_j^{-p}}\right] \qquad (7.6)$$

where K is a constant, $W(t_{ij}) = 1 - \exp\left(\frac{-t_{ij}}{\beta}\right)$ & $W(t_0) = 1 - \exp\left(\frac{-t_0}{\beta}\right)$

The first derivative of the logarithm of the likelihood function in equation (7.6) with respect to β, C & p are obtained as:

$$\frac{\partial InL}{\partial \beta} = -\sum_{j=1}^{k} \frac{r_j}{\beta} + \sum_{j=1}^{k} \sum_{i=1}^{r_j} \frac{t_{ij}}{\beta^2} - \sum_{j=1}^{k} \sum_{i=1}^{r_j} \frac{(CV_j^{-p} - 1)}{W(t_{ij})} \frac{t_{ij}}{\beta^2} \exp\left(\frac{-t_{ij}}{\beta}\right) + \sum_{j=1}^{k} \frac{(n_j - r_j)}{\varphi(t_0)} \frac{t_0 CV_j^{-p}}{\beta^2} (W(t_0))^{CV_j^{-p} - 1} \exp\left(-\frac{t_0}{\beta}\right) \qquad (7.7)$$

$$\frac{\partial InL}{\partial C}=\sum_{j=1}^{k}\frac{r_j}{C}+\sum_{j=1}^{k}\sum_{i=1}^{r_j}V_j^{-p}InW\left(t_{ij}\right)-\sum_{j=1}^{k}\frac{\left(n_j-r_j\right)}{\varphi\left(t_0\right)}V_j^{-p}\left(W\left(t_0\right)\right)^{CV_j^{-p}}In\left(W\left(t_0\right)\right) \quad (7.8)$$

$$\frac{\partial InL}{\partial p}=\sum_{j=1}^{k}r_jInV_j-\sum_{j=1}^{k}\sum_{i=1}^{r_j}CV_j^{-p}InV_jIn\left(W\left(t_{ij}\right)\right)+\sum_{j=1}^{k}\frac{\left(n_j-r_j\right)}{\varphi\left(t_0\right)}CV_j^{-p}InV_jInW\left(t_0\right)\left(W\left(t_0\right)\right)^{CV_j^{-p}} \quad (7.9)$$

Where, $\phi(t_0)=1-(W(t_0))^{CV_j^{-p}}$

The ML estimates of β, C and p are obtained by equating the above equations to zero. Also, the variance-covariance matrix is obtained using the fisher information matrix of the form:

$$I=\begin{bmatrix}\frac{-\partial^2 InL}{\partial\beta^2} & \frac{-\partial^2 InL}{\partial C\partial\beta} & \frac{-\partial^2 InL}{\partial p\partial\beta}\\ \frac{-\partial^2 InL}{\partial\beta\partial C} & \frac{-\partial^2 InL}{\partial C^2} & \frac{-\partial^2 InL}{\partial P\partial C}\\ \frac{-\partial^2 InL}{\partial\beta\partial p} & \frac{-\partial^2 InL}{\partial C\partial p} & \frac{-\partial^2 InL}{\partial p^2}\end{bmatrix} \quad (7.10)$$

Where,

$$\frac{\partial^2 InL}{\partial\beta^2}=\sum_{j=1}^{k}\frac{r_j}{\beta^2}-2\sum_{j=1}^{k}\sum_{i=1}^{r_j}\frac{t_{ij}}{\beta^3}-\sum_{j=1}^{k}\sum_{i=1}^{r_j}\left[\left(CV_j^{-p}-1\right)te^{\frac{t_{ij}}{\beta}}\frac{2\beta e^{\frac{t_{ij}}{\beta}}-2\beta-t_{ij}}{\beta^4\left(e^{\frac{t_{ij}}{\beta}}-1\right)^2}\right]+\sum_{j=1}^{k}\left(n_j-r_j\right)t_0CV_j^{-p}$$

$$\left[\frac{\left(W\left(t_0\right)\right)^{CV_j^{-p}}\left\{\left(\left(2V_j^p\beta^3-t_0V_j^p\beta^2\right)\left(W\left(t_0\right)\right)^{CV_j^{-p}}+t_0V_j^p\right)e^{\frac{t_0}{p}}-2V_j^p\beta^3\left(W\left(t_0\right)\right)^{CV_j^{-p}}-Ct_0\right\}}{V_j^p\left\{\beta^2\left(W\left(t_0\right)\right)CV_j^{-p}-1\right\}^2\left(e^{\frac{t}{\beta}}-1\right)^2}\right] \quad (7.11)$$

$$\frac{\partial^2 InL}{\partial C^2}=-\sum_{j=1}^{k}\frac{r_j}{C^2}-\sum_{j=1}^{k}\left(n_j-r_j\right)V_j^{-p}\left(W\left(t_0\right)\right)^{CV_j^{-p}}\left\{In\left(W\left(t_0\right)\right)\right\}^2\left\{\frac{\varphi\left(t_0\right)V_j^{-p}+\left(W\left(t_0\right)\right)^{CV_j^{-P}}V_j^{-p}}{\left(\varphi\left(t_0\right)\right)^2}\right\} \quad (7.12)$$

$$\frac{\partial^2 InL}{\partial p^2}=\sum_{j=1}^{k}\sum_{i=1}^{r_j}CV_j^{-p}\left(InV_j\right)^2InW\left(t_{ij}\right)+$$

$$\sum_{j=1}^{k}\left(n_j-r_j\right)InV_jCV_j^{-2p}InW\left(t_0\right)\left[\frac{\left(W\left(t_0\right)\right)^{CV_j^{-p}}InV_j\left\{\phi\left(t_0\right)V_j^{-p}+CInW\left(t_0\right)\right\}}{\left(\phi\left(t_0\right)\right)^2}\right] \quad (7.13)$$

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$$\frac{\partial^2 InL}{\partial\beta\partial C}=-\sum_{j=1}^{k}\sum_{i=1}^{r_j}\frac{t_{ij}}{\beta^2}e^{-\frac{t_{ij}}{\beta}}+\sum_{j=1}^{k}(n_j-r_j)t_0(W(t_0))^{CV_j^{-p}}\left(\frac{CInW(t_0)+V_j^p}{\phi(t_0)\beta^2\left(e^{\frac{t_0}{\beta}}-1\right)}\right) \quad (7.14)$$

$$\frac{\partial^2 InL}{\partial p\partial C}=-\sum_{j=1}^{k}\sum_{i=1}^{r_j}V_j^{-p}InW(t_{ij})InV_j-\sum_{j=1}^{k}(n_j-r_j)InW(t_0)\left[(W(t_0))^{CV_j^{-p}}\left(\frac{CInW(t_0)-\phi(t_0)}{\phi(t_0)V_j^{2p}}\right)\right] \quad (7.15)$$

$$\frac{\partial^2 InL}{\partial\beta\partial p}=\sum_{j=1}^{k}\sum_{i=1}^{r_j}\frac{t_{ij}}{\beta^2}\frac{InV_j}{W(t_{ij})}e^{\frac{-t_{ij}}{\beta^2}}+\sum_{j=1}^{k}t_0\frac{(n_j-r_j)}{V_j^p\beta^2}CV_j^{-p}InV_j(W(t_0))^{CV_j^{-p}}\left[\frac{CInW(t_0)+V_j^p\phi(t_0)}{(\phi(t_0))^2\left(e^{t_0/\beta}-1\right)}\right] \quad (7.16)$$

Thus, the approximate $(1-\lambda)100\%$ confidence intervals for $\alpha, C\ \&\ p$ are given by:

$$\hat{\beta}\pm Z_{\lambda/2}\sqrt{\operatorname{var}(\hat{\beta})},\qquad \hat{C}\pm Z_{\lambda/2}\sqrt{\operatorname{var}(\hat{C})},\qquad \hat{p}\pm Z_{\lambda/2}\sqrt{\operatorname{var}(\hat{p})},$$

Where $Z_{\lambda/2}$ is the $100(1-\lambda/2)$ percentile of a standard normal variate.

Simulation Procedure

Numerical studies are carried out to check the performances of the ML estimates. The invariance property of MLE is used to estimate the MLEs of shape Parameter β_j through;

$$\alpha_j=CV_j^{-p};\ \ C>0, p>0,\ \ j=1,2,\dots,k$$

The detailed steps are given below

1) The total of thousand random samples of sizes 50, 100, 150 and 200 are generated from Generalised Exponential distribution.
2) Three different levels of stress, k=3, are chosen as below. $(V_1=1, V_2=1.5, V_3=2), n_j=\dfrac{n}{3}\ \&\ r_j=60\%\, n_j$.
3) For sample sizes, type-I censored samples are used to estimate the parameters using Newton-Raphson method.
4) The RABs and MSE are tabulated for all sets of (β_0, C_0, p_0).
5) Using the invariance property of MLEs, we calculate the MLEs of the shape parameter α_u at usual stress level $V_u=0.5$. We also calculate the reliability function for different values of $\beta, C, p\ \&\ t_0$,

$$\hat{R}_u(t_0) = 1 - \left(1 - e^{-\frac{t}{\beta_0}}\right)^{\alpha_0}$$

Also, at each at mission time $(t_0 = 1.5, 1.8 \ \& \ 2.2)$, the MLEs of the reliability function are predicted for all sets of parameters.

Simulation results are summarised in Tables (7.1), (7.2) and (7.3). Tables (7.1) and (7.2) give the estimators, RABs and MSEs. Tables (7.3) give the estimated shape parameter under $V_u = 0.5$. And the reliability function is predicted under $V_u = 0.5$.

Table 7.1: *The Estimates, Relative Bias and MSE of the parameters* $(\beta, C, P, \alpha_1, \alpha_2, \alpha_3)$ *under type-II censoring*

n	Parameters	$(\beta_0 = 0.25, C_0 = 1.5, p_0 = 1)$			$(\beta_0 = 1, C_0 = 1.5, p_0 = 1)$		
		Estimator	RABs	MSEs	Estimator	RABs	MSEs
50	β	0.229	0.084	0.078	0.930	0.070	0.076
	C	1.411	0.059	0.044	1.421	0.052	0.063
	P	0.930	0.070	0.062	0.925	0.075	0.071
	α_1	1.411	0.059	0.044	1.421	0.052	0.063
	α_2	0.967	0.057	0.042	0.976	0.050	0.062
	α_3	0.740	0.056	0.042	0.748	0.049	0.059
100	β	0.234	0.064	0.064	0.947	0.053	0.056
	C	1.429	0.047	0.035	1.436	0.042	0.048
	P	0.933	0.067	0.071	1.098	0.098	0.077
	α_1	1.429	0.047	0.035	1.436	0.042	0.048
	α_2	0.955	0.045	0.034	0.920	0.040	0.046
	α_3	0.748	0.044	0.033	0.670	0.039	0.045
150	β	0.239	0.044	0.042	0.951	0.049	0.051
	C	1.449	0.034	0.034	1.561	0.040	0.041
	P	0.946	0.054	0.046	1.052	0.052	0.046
	α_1	1.449	0.034	0.034	1.561	0.040	0.041
	α_2	0.987	0.033	0.033	1.018	0.039	0.040
	α_3	0.752	0.032	0.032	0.752	0.038	0.039

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200	β	0.252	0.008	0.010	0.976	0.024	0.030
	C	1.480	0.013	0.003	1.467	0.022	0.023
	P	1.113	0.113	0.010	0.955	0.045	0.031
	α_1	1.480	0.013	0.003	1.467	0.022	0.023
	α_2	0.942	0.012	0.002	0.996	0.021	0.023
	α_3	0.684	0.012	0.003	0.756	0.021	0.022

Table 7.2: *The Estimates, Relative Bias and MSE of the parameters* $(\beta, C, P, \alpha_1, \alpha_2, \alpha_3)$ *under type-II censoring*

N	parameters	$(\beta_0 = 0.25, C_0 = 1, p_0 = 1)$			$(\beta_0 = 1, C_0 = 1, p_0 = 1.5)$		
		Estimator	RABs	MSEs	Estimator	RABs	MSEs
50	β	0.231	0.076	0.081	0.928	0.072	0.067
	C	1.103	0.103	0.047	1.071	0.071	0.067
	P	0.929	0.071	0.058	1.421	0.052	0.063
	α_1	1.103	0.103	0.047	1.071	0.071	0.067
	α_2	0.756	0.074	0.046	0.601	0.069	0.065
	α_3	0.579	0.072	0.045	0.400	0.068	0.064
100	β	0.237	0.052	0.057	0.939	0.061	0.055
	C	1.098	0.098	0.042	1.056	0.044	0.050
	P	0.937	0.063	0.075	1.436	0.042	0.065
	α_1	1.098	0.098	0.042	1.056	0.044	0.050
	α_2	0.750	0.095	0.040	0.590	0.043	0.049
	α_3	0.573	0.093	0.039	0.390	0.043	0.048
150	β	0.239	0.044	0.045	0.948	0.052	0.042
	C	1.049	0.049	0.033	1.041	0.041	0.036
	P	0.958	0.042	0.040	1.561	0.040	0.056
	α_1	1.049	0.049	0.033	1.041	0.041	0.036
	α_2	0.711	0.048	0.032	0.552	0.040	0.035
	α_3	0.539	0.047	0.032	0.352	0.039	0.034

200	β	0.252	0.008	0.017	0.981	0.019	0.036
	C	1.060	0.060	0.023	1.035	0.025	0.031
	P	1.013	0.013	0.021	1.467	0.022	0.025
	α_1	1.060	0.060	0.023	1.035	0.025	0.031
	α_2	0.702	0.060	0.022	0.570	0.024	0.030
	α_3	0.525	0.059	0.022	0.374	0.024	0.030

Table 7.3: *The estimated shape parameter and Reliability function at normal stress level taking n=200.*

β_0	C_0	P_0	α_0	t_0	$R_u(t_0)$
0.25	1.5	1	3.201165	0.2 0.4 0.6	0.8518 0.5141 0.2624
1	1.5	1	2.843896	0.2 0.4 0.6	0.9922 0.9573 0.8959
0.25	1	1	2.139189	0.2 0.4 0.6	0.7208 0.3826 0.1840
1	1	1.5	2.861221	0.2 0.4 0.6	0.9924 0.9581 0.8974

The Replacement Policy with the prospective of Pro-Rata Rebate Warranty Scheme

This warranty policy is applicable on the non-repairable products. The product is replaced upon failure or at a certain time age (τ), which among the two occurs first. Upon failure at $t \le \tau$, a failure replacement is performed with $Cd > 0$ (downtime cost) and $Cp > 0$ (purchasing cost). The customer is refunded a proportion of sales price Cp if the defect/failure occurs in the warranty period (w). The rebate function is given by:

$$R(t) = \begin{cases} Cp\left(1 - \frac{t}{w}\right) & 0 \le t \le w \\ 0 & t > w \end{cases} \tag{7.17}$$

There is some literature available on age-replacement policy, e.g. Chien and Chen (2007a), Chien and Chen (2007b), Huang et al. (2008), Chien (2010), Chien at al. (2014), Na and Sheng

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(2014), have used the different warranty policies and observed their effects under both producer and consumer perspective. The current study dealt with estimating the expected total cost and expected cost rate for age replacement of units under warranty policy. The pro-rata rebate warranty policy has also been taken into consideration. It is assumed that there is no salvage value for the preventively replaced product. The preventive replacement is carried out with cost Cp at the product age τ.

Therefore, the total cost incurred in a renewal cycle is:

$$C(d) = \begin{cases} Cd + Cp - R(t) & 0 \le t \le w \\ Cd + Cp & w < t < \tau \\ Cp & t \ge \tau \end{cases} \tag{7.18}$$

According to Chein (2010) and Chein et.al (2014), the expected total cost function under this policy is:

$$E(C(t)) = CdF(\tau) + Cp\frac{\int_0^w \bar{F}(u)du}{w} \tag{7.19}$$

And, the expected cost rate is

$$E(CR(t)) = \frac{E(C(t))}{\int_0^\tau \bar{F}(u)du} \tag{7.20}$$

Where $\int_0^\tau \bar{F}(u)du$ is the expected cycle time which is denoted by $E(T(\tau))$.

Under Generalised Exponential distribution:

We have

$$F(u) = \left(1 - e^{-\frac{u}{\beta}}\right)^\alpha ; \; u > 0, \alpha, \beta > 0 \tag{7.21}$$

Therefore z

Also,

$$\int_0^\tau \bar{F}(u)du = \tau - \int_0^\tau \left(1 - e^{-\frac{u}{\beta}}\right)^\alpha du \tag{7.23}$$

Substituting equations (7.22) and (7.23) into equations (7.19) and (7.20), respectively, we obtain the expected total cost and expected cost rate for the non-repairable product. It can be seen that the function defined in the above equations does not have the elementary integral. Therefore, a numerical approximation can be obtained by substituting the values of all the parameters involved, except for the variable of integration.

Now for an application example, if the item is replaced with downtime cost $Cd = 50$ and purchasing cost $Cp = 1000$. The expected total cost, expected cost rate, and expected cycle time are estimated. Also, the estimated values of the parameters of generalised exponential distribution α and β are obtained under normal conditions as shown in table 7.4.

Table 7.4: *The expected total cost, the expected cycle time and the expected cost rate for age-replacement under warranty policy on Generalised Exponential distribution.*

B	A	w	τ	E(C(τ))	E(T(τ))	CR(τ)
2	0.2	5	7	930.1232	5.7921	230.885
3	0.2	5	7	944.1763	6.4909	215.4057
4	0.2	5	7	955.8282	6.9264	199.7983
5	0.2	5	7	973.2882	7.3589	174.5714
5	0.3	5	7	882.6932	6.0825	183.0282
5	0.4	5	7	830.1848	5.2488	204.6751
5	0.4	6	7	903.7913	6.2867	208.0381
5	0.4	7	7	947.5114	7.5112	218.3333
5	0.4	8	8	988.8976	8.0751	203.3333
5	0.4	8	9	1012.512	8.4758	200.4356
5	0.4	8	10	1050.812	8.8752	216.6667

Results and Conclusion

In the table (7.1) and (7.2), it can observe that the modules of the difference between the true value of the parameter and its estimator converges to zero, hence consistent. From table (7.3), it can be noticed that the reliability function decreases as the mission time t_0 increases. It is

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obvious that whenever a product is tested for a long period of time, its reliability decreases because of the wear out in the product. The studies show that the preventive replacement will be strongly affected under PRRW. Particularly, when the product is proven to failures, and adding the PRRW will extend the optimal replacement age closer to the warranty period.

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