Chapter 6:

Bayesian Inference for Exponential Rayleigh Distribution Using R Software

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Introduction

Exponential-Rayleigh (ER) distribution is a newly proposed lifetime model introduced and discussed by Kawsar and Ahmad (2017). It is a versatile distribution and can take a variety of shapes such as positively skewed, reversed-J and tends to be symmetric. Exponential-Rayleigh distribution is a continuous distribution with wide range of applications in reliability fields and is used for modelling lifetime phenomena. The cdf and pdf of the ERD are given as

$$f(x)=\lambda\beta x e^{\frac{\beta}{2}x^2} e^{-\lambda\left(e^{\frac{\beta}{2}x^2}-1\right)}; \qquad x,\lambda,\beta>0\ , \tag{6.1}$$

$$F(x)=1-e^{-\lambda\left(e^{\frac{\beta}{2}x^2}-1\right)} \tag{6.2}$$

The main purpose of this chapter is to study the Bayesian approach for the parameter of ER distribution. There are numerous good sources which provide the detailed explanation of Bayesian approach while then, a number of authors have studied and obtained various probability distributions based on the Estimation of the Bayesian approach. Ahmed et al. (2007) discussed the exponential distribution (ED) from a Bayesian point of view. James Dow (2015) obtained the Bayesian Inference for the parameter of Weibull-Pareto distribution. Naqash et al. (2016) studied Bayesian Analysis of Generalized Exponential Distribution while as Kawsar and Ahmad (2017) considered the estimation for the parameter of Weibull- Rayleigh (WR) distribution. They obtained Baye's estimators for the parameter of WR distribution by using different Informative and Non-Informative priors under different symmetric and asymmetric loss functions. They also compared the classical method with Bayesian method by using mean square error through simulation study with varying sample sizes.

Parameter Estimation

Consider a random sample $x_1, x_2, x_3, \ldots x_n$ having density function of (6.1) and then the likelihood function of the given distribution is as follows:

$$L(x) = \lambda^n \beta^n \prod_{i=1}^{n} \left\{ x_i e^{\frac{\beta}{2} x_i^2} e^{-\lambda \left(e^{\frac{\beta}{2} x_i^2} - 1 \right)} \right\}. \tag{6.3}$$

The corresponding log likelihood function of the equation (6.3) is given as under:

$$\log\ L(x) = n \log \lambda + n \log \beta + \sum_{i=1}^{n} \log x_i + \frac{\beta}{2} \sum_{i=1}^{n} x_i^{\,2} - \lambda \sum_{i=1}^{n} \left(e^{\frac{\beta}{2} x_i^2} - 1 \right). \tag{6.4}$$

Differentiating (6.4) with respect to λ, when the parameter β is assumed to be known, then the MLE is obtained as

$$\frac{\partial}{\partial \lambda} \log L(x) = 0, \Rightarrow \hat{\lambda} = \frac{n}{\sum_{i=1}^{n} \left(e^{\frac{\beta}{2} x_i^2} - 1 \right)}. \tag{6.5}$$

Posterior Distribution and Baye's Estimators under Non-Informative Prior Using Different Loss Functions:

The extended Jeffrey's prior suggested by Al-Kutubi (2005) is given as

$$g_1(\lambda) = \frac{1}{\lambda^{2c_1}};\ c_1 \in R^+ \tag{6.6}$$

Combining the likelihood function (6.3) and the above prior distribution, then the posterior density of λ is derived as follows:

$$g_1(\lambda \mid x) = \frac{T_1^{\,n-2c_1+1}}{\Gamma(n - 2c_1 + 1)} \lambda^{n-2c_1}\ e^{-\lambda T_1} \quad ; \quad \lambda > 0\ . \tag{6.7}$$

Hence the posterior density of $g_1(\lambda \mid x) \sim G((n - 2c_1 + 1), T_1)$, where $T_1 = \sum_{i=1}^{n} \left(e^{\frac{\beta}{2} x_i^2} - 1 \right)$

With the above prior, we use three different loss functions namely Al-Bayyati's loss function (ABLF), Entropy loss function (ELF) and LINEX loss function (LLF) to find Bayes estimates for the parameter of model (6.1).

Under ABLF the risk function is given by

$$R(\hat{\lambda},\lambda)=\int_0^\infty \lambda^{c_2}(\hat{\lambda}-\lambda)^2\,\frac{T_1^{\,n-2c_1+1}}{\Gamma(n-2c_1+1)}\lambda^{n-2c_1}\;e^{-\lambda T_1}d\lambda \tag{6.8}$$

On solving (6.8), we get

$$R(\hat{\lambda},\lambda)=\frac{1}{\Gamma n-2c_1+1)}\left[\frac{\hat{\lambda}^2\Gamma(n-2c_1+c_2+1)}{T_1^{\,c_2}}+\frac{\Gamma(n-2c_1+c_2+3)}{T_1^{\,c_2+2}}-\frac{2\hat{\lambda}\Gamma(n-2c_1+c_2+2)}{T_1^{\,c_2+1}}\right].$$

Minimization of the risk with respect to $\hat{\lambda}$ gives us the Bayes estimator:

$$\hat{\lambda}_{ABLF}=\frac{(n-2c_1+c_2+1)}{T_1}\;;\text{ where } T_1=\sum_{i=1}^{n}\left(e^{\frac{\beta}{2}x_i^2}-1\right)\;. \tag{6.9}$$

Remark 6.1:

Replacing $c_2=0$ and $c_1=1/2$ in (6.9) we get the same Bayes estimator as obtained under SELF using the Jeffrey's prior, replace $c_2=0$ and $c_1=3/2$ we get the same Bayes estimator as obtained under SELF using Hartigan's prior and replace $c_2=0$ and $c_1=0$ we get the we get the same Bayes estimator as obtained under SELF using Uniform prior. Under ELF the risk function is given by

$$R(\hat{\lambda},\lambda)=b_1\int_0^\infty\left[\frac{\hat{\lambda}}{\lambda}-\log\left(\frac{\hat{\lambda}}{\lambda}\right)-1\right]\;\frac{T_1^{\,n-2c_1+1}}{\Gamma(n-2c_1+1)}\lambda^{n-2c_1}\;e^{-\lambda T_1}\;\;d\lambda\;. \tag{6.10}$$

On solving (6.10), we get

$$R(\hat{\lambda},\lambda)=b_1\left[\hat{\lambda}\frac{T_1}{(n-2c_1)}-\log(\hat{\lambda})+\frac{\Gamma'(n-2c_1+1)}{\Gamma(n-2c_1+1)}-1\right].$$

Minimization of the risk with respect to $\hat{\lambda}$ gives us the Bayes estimator:

$$\hat{\lambda}_{ELF}=\frac{(n-2c_1)}{T_1}\;;\text{ where } T_1=\sum_{i=1}^{n}\left(e^{\frac{\beta}{2}x_i^2}-1\right)\;. \tag{6.11}$$

Remark 6.2:

Replacing $c_1 = 1/2$ in (6.11) we get the same Bayes estimator as obtained under the Jeffrey's prior, replace $c_1 = 3/2$ we get the Hartigan's prior and replace $c_1 = 0$ we get the Uniform prior.

Under LLF the risk function is given by

$$R(\hat{\lambda},\lambda) = \int_0^\infty \left(\exp\left\{b_2\left(\hat{\lambda}-\lambda\right)\right\} - b_2\left(\hat{\lambda}-\lambda\right) - 1\right) \frac{T_1^{\,n-2c_1+1}}{\Gamma(n-2c_1+1)} \lambda^{n-2c_1} \; e^{-\lambda T_1} d\lambda. \qquad (6.12)$$

On solving (6.12), we get

$$R(\hat{\lambda},\lambda) = \frac{T_1^{\,n-2c_1+1}}{\Gamma(n-2c_1+1)} \left[\begin{aligned} & e^{b_2\hat{\lambda}} \frac{\Gamma(n-2c_1+1)}{(b_2+T_1)^{n-2c_1+1}} - b_2\hat{\lambda} \frac{\Gamma(n-2c_1+1)}{T_1^{\,n-2c_1+1}} \\ & \qquad + b_2 \frac{\Gamma(n-2c_1+2)}{T_1^{\,n-2c_1+2}} - \frac{\Gamma(n-2c_1+1)}{T_1^{\,n-2c_1+1}} \end{aligned} \right]$$

$$R(\hat{\lambda},\lambda) = e^{b_2\hat{\lambda}} \left(\frac{T_1}{b_2+T_1}\right)^{n-2c_1+1} - b_2\hat{\lambda} + b_2 \frac{(n-2c_1+1)}{T_1} - 1.$$

Minimization of the risk with respect to $\hat{\lambda}$ gives us the Bayes estimator:

$$\hat{\lambda}_{LLF} = \frac{1}{b_2} \log\left(\frac{b_2+T_1}{T_1}\right)^{n-2c_1+1} ; \text{where } T_1 = \sum_{i=1}^{n} \left(e^{\frac{\beta}{2}x_i^2} - 1\right). \qquad (6.13)$$

Remark 6.3:

If we put $c_1 = 1/2$ in (6.13) we get the same Bayes estimator as obtained under the Jeffrey's prior, If $c_1 = 3/2$ we get the Hartigan's prior and If $c_1 = 0$ we get the Uniform prior.

Posterior Distribution and Baye's Estimators under Informative Prior Using Different Loss Functions:

The gamma distribution is used as an informative prior with hyper parameters a *and* b, having the following p.d.f as:

$$g_2(\lambda) \propto \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}; \quad 0 < \lambda < \infty, a, b > 0. \qquad (6.14)$$

Combining the likelihood function (6.3) and the prior distribution (6.14), then the posterior density of λ is derived as follows:

$$g_2(\lambda \mid x) = \frac{T_2^{\,n+a}}{\Gamma(n+a)} \lambda^{n+a-1} \, e^{-\lambda T_2} \quad ; \quad \lambda > 0 \, . \tag{6.15}$$

Hence the posterior density of $g_2(\lambda \mid x) \sim G((n+a), T_2)$; where $T_2 = b + \sum_{i=1}^{n} \left(e^{\frac{\beta}{2} x_i^2} - 1 \right)$.

Remark 6.4:

For $a = b = 0$ in (6.15), the posterior distribution under the gamma prior reduces to posterior distribution under the Jeffrey's prior.
For $a = 1, b = 0$ in (6.15), the posterior distribution under the gamma prior reduces to posterior distribution under the Uniform prior.

Under ABLF the risk function is given by

$$R(\hat{\lambda}, \lambda) = \int_0^\infty \lambda^{c_2} (\hat{\lambda} - \lambda)^2 \, \frac{T_2^{\,n+a}}{\Gamma(n+a)} \lambda^{n+a-1} \, e^{-\lambda T_2} d\lambda \tag{6.16}$$

On solving (3.16), we get

$$R(\hat{\lambda}, \lambda) = \frac{1}{\Gamma n + a)} \left[\frac{\hat{\lambda}^2 \Gamma(n + c_2 + a)}{T_2^{\,c_2}} + \frac{\Gamma(n + a + c_2 + 1)}{T_2^{\,c_2+2}} - \frac{2\hat{\lambda}\Gamma(n + a + c_2 + 1)}{T_2^{\,c_2+1}} \right].$$

Minimization of the risk with respect to $\hat{\lambda}$ gives us the Bayes estimator:

$$\hat{\lambda}_{ABLF} = \frac{(n + a + c_2)}{T_2} \text{ ; where } T_2 = b + \sum_{i=1}^{n} \left(e^{\frac{\beta}{2} x_i^2} - 1 \right) \, . \tag{6.17}$$

Remark 6.5:

Replacing $c_2 = 0$ in (3.17), we get the same Bayes estimator as obtained under the SELF.

Under ELF the risk function is given by

$$R(\hat{\lambda}, \lambda) = b_1 \int_0^\infty \left[\frac{\hat{\lambda}}{\lambda} - \log\left(\frac{\hat{\lambda}}{\lambda} \right) - 1 \right] \frac{T_2^{\,n+a}}{\Gamma(n+a)} \lambda^{n+a-1} \, e^{-\lambda T_2} d\lambda \, . \tag{6.18}$$

On solving (6.18), we get

$$R(\hat{\lambda},\lambda)=b_1\left[\hat{\lambda}\frac{T_2}{(n+a-1)}-\log(\hat{\lambda})+\frac{\Gamma'(n+a)}{\Gamma(n+a)}-1\right].$$

Minimization of the risk with respect to $\hat{\lambda}$ gives us the Bayes estimator:

$$\hat{\lambda}_{ELF}=\frac{(n+a-1)}{T_2}\quad ;\text{where } T_2=b+\sum_{i=1}^{n}\left(e^{\frac{\beta}{2}x_i^2}-1\right). \tag{6.19}$$

Under LLF the risk function is given by

$$R(\hat{\lambda},\lambda)=\int_0^\infty\left(\exp\left\{b_2\left(\hat{\lambda}-\lambda\right)\right\}-b_2\left(\hat{\lambda}-\lambda\right)-1\right)\frac{T_2^{\,n+a}}{\Gamma(n+a)}\lambda^{n+a-1}\;e^{-\lambda T_2}d\lambda \tag{6.20}$$

On solving (6.20), we get

$$R(\hat{\lambda},\lambda)=\frac{T_2^{\,n+a}}{\Gamma(n+a)}\left[\begin{array}{r}e^{b_2\hat{\lambda}}\dfrac{\Gamma(n+a)}{(b_2+T_2)^{n+a}}-b_2\hat{\lambda}\dfrac{\Gamma(n+a)}{T_2^{\,n+a}}\\ +b_2\dfrac{\Gamma(n+a+1)}{T_2^{\,n+a+1}}-\dfrac{\Gamma(n+a)}{T_2^{\,n+a}}\end{array}\right]$$

$$R(\hat{\lambda},\lambda)=e^{b_2\hat{\lambda}}\left(\frac{T_2}{b_2+T_2}\right)^{n+a}-b_2\hat{\lambda}+b_2\frac{(n+a)}{T_2}-1.$$

Minimization of the risk with respect to $\hat{\lambda}$ gives us the Bayes estimator:

$$\hat{\lambda}_{LLF}=\frac{1}{b_2}\log\left(\frac{b_2+T_2}{T_2}\right)^{n+a}\quad ;\text{where } T_2=b+\sum_{i=1}^{n}\left(e^{\frac{\beta}{2}x_i^2}-1\right). \tag{6.21}$$

Simulation Study

In the simulation study, three samples of sizes 25, 50 and 100 to signify small, medium, and large data sets have been generated from R software to examine the performance of Classical and Bayesian estimates for the parameter of Exponential-Rayleigh (ER) distribution under different priors using different loss functions. The data sets are obtained by using the inverse cdf method and the value of the parameters α & λ are chosen as $\alpha=0.5$ and $\lambda=0.5, 1.0$ & 1.5. The values of Jeffrey's extension were $c_1=(0.4,1.4)$ and the values of hyper parameters were a= (0.4, 1.4) and b= (0.4, 1.4). The results are replicated 1000 times and the average results are presented in table 6.1 and table 6.2.

Table 6.1: *Baye's estimators and MSE (in parenthesis) under Extension of Jeffery's prior*

n	β	λ	c_1	$\hat{\lambda}_{ML}$	$\hat{\lambda}_{ABLF}$		$\hat{\lambda}_{ELF}$	$\hat{\lambda}_{LLF}$	
					c_2=0.3	c_2=-0.3		b_2=0.4	b_2=-0.4
25	0.5	0.5	0.4	0.44148 (0.0887)	0.45031 **(0.01033)**	0.43972 (0.0114)	0.42736 (0.0131)	0.44346 (0.0111)	0.44659 (0.01072)
			1.4	0.44148 (0.0887)	0.41499 **(0.01445)**	0.40439 (0.0163)	0.39204 (0.0188)	0.40826 (0.0156)	0.41115 (0.01513)
		1.0	0.4	0.88297 (0.4342)	0.90063 **(0.04131)**	0.87944 (0.0459)	0.85471 (0.0525)	0.88380 (0.0449)	0.89638 (0.04217)
			1.4	0.88297 (0.4342)	0.82999 **(0.05784**)	0.80879 (0.0654)	0.78408 (0.0755)	0.81366 (0.0636)	0.82524 (0.06366)
		1.5	0.4	1.32445 (1.1814)	1.35094 **(0.09295)**	1.31916 (0.1034)	1.28207 (0.1182)	1.32110 (0.1027)	1.34939 (0.09341)
			1.4	1.32445 (1.1810)	1.24498 (**0.13015**)	1.21319 (0.1473)	1.176115 (0.1700)	1.21625 (0.1456)	1.24230 (0.13152)
50	0.5	0.5	0.4	0.4887 (0.0168)	0.49356 **(0.00484)**	0.48769 (0.0049)	0.48085 (0.0051)	0.48967 (0.0049)	0.49159 (0.00487)
			1.4	0.4887 (0.0168)	0.47401 (0.00528)	0.46815 (0.0056)	0.46131 (0.0061)	0.47016 (0.0054)	0.47200 (0.00539)
		1.0	0.4	0.97735 (0.0605)	0.98712 **(0.01935)**	0.97539 (0.0197)	0.96171 (0.0206)	0.97744 (0.0196)	0.98511 (0.01940)
			1.4	0.97735 (0.0605)	0.94803 (**0.02112)**	0.93629 (0.0224)	0.92262 (0.0244)	0.93849 (0.0221)	0.94587 (0.02135)
		1.5	0.4	1.4662 (0.1519)	1.48068 **(0.04353)**	1.46309 (0.0445)	1.44256 (0.0464)	1.46332 (0.0445)	1.48058 (0.04353)
			1.4	1.46602 (0.1519)	1.42204 **(0.04751)**	1.40445 (0.0505)	1.38392 (0.0549)	1.40502 (0.0504)	1.42159 (0.04758)
100	0.5	0.5	0.4	0.51758 (0.0083)	0.51344 **(0.00286)**	0.51706 (0.0029)	0.52016 (0.0030)	0.518075 (0.0030)	0.51915 (0.00305)
			1.4	0.51758 (0.0083)	0.50308 (**0.00264**)	0.50671 (0.0026)	0.50981 (0.0027)	0.50773 (0.0026)	0.50879 (0.00271)
		1.0	0.4	1.03515 (0.0329)	1.02687 **(0.01146)**	1.03412 0.01190	1.04033 (0.0123)	1.03508 0.01197	1.039375 (0.01229)
			1.4	1.03515 (0.0329)	1.00617 **(0.01056)**	1.01341 (0.0107)	1.01962 (0.0109)	1.01442 (0.0107)	1.01863 (0.01087)
		1.5	0.4	1.55273 (0.0730)	**0.51344** **(0.00286)**	1.55117 0.0267	1.56049 (0.0278)	1.55102 (0.0267)	1.56068 (0.02784)
			1.4	1.55273 (0.0730)	0.50308 (**0.00264**)	1.52012 (0.0240)	1.52944 (0.0245)	1.52006 (0.0240)	1.52953 (0.02455)

Table 6.2: Baye’s estimators and MSE (in parenthesis) under Gamma prior

n	β	λ	$a=b$	$\hat{\lambda}_{ML}$	$\hat{\lambda}_{ABLF}$		$\hat{\lambda}_{ELF}$	$\hat{\lambda}_{LLF}$	
					c_2=0.3	c_2=-0.3		b_2=0.4	b_2=-0.4
25	0.5	0.5	0.4	0.44148 (0.08879)	0.45066 **(0.0102)**	0.44014 (0.01139)	0.42786 (0.01301)	0.44384 (0.01096)	0.44697 (0.0106)
			1.4	0.44148 (0.08879)	0.46013 **(0.0094)**	0.44979 (0.01036)	0.43773 (0.01173)	0.45339 (0.01001)	0.45654 (0.0097)
		1.0	0.4	0.88297 (0.43422)	0.89505 **(0.0418)**	0.874151 (0.04665)	0.84977 (0.05338)	0.87849 (0.04557)	0.89082 (0.0427)
			1.4	0.88297 (0.43422)	0.89858 **(0.0401)**	0.87839 (0.04469)	0.85483 (0.05098)	0.88256 (0.04369)	0.89452 (0.0410)
		1.5	0.4	1.32445 (1.18140)	1.33328 **(0.0961)**	1.30215 (0.10750)	1.26584 (0.12319)	1.30423 (0.10669)	1.33158 (0.0967)
			1.4	1.32445 (1.18140)	1.31685 **(0.0977)**	1.28725 (0.10948)	1.25273 (0.12536)	1.28937 (0.10858)	1.31506 (0.0984)
50	0.5	0.5	0.4	0.48867 (0.01628)	0.49358 **(0.0048)**	0.48774 (0.00493)	0.48093 (0.00514)	0.48971 (0.00489)	0.49162 (0.0048)
			1.4	0.48867 (0.01628)	0.49847 **(0.0047)**	0.49268 (0.00483)	0.48593 (0.00498)	0.49462 (0.00481)	0.49653 (0.0047)
		1.0	0.4	0.97735 (0.06605)	0.98334 **(0.0192)**	0.97170 (0.01976)	0.95813 (0.02073)	0.97375 (0.01965)	0.98133 (0.0193)
			1.4	0.97735 (0.06605)	0.98366 **(0.0188)**	0.97224 (0.01937)	0.95892 (0.02029)	0.97425 (0.01927)	0.98169 (0.0189)
		1.5	0.4	1.46602 (0.15194)	1.46931 **(0.0432)**	1.45193 (0.04464)	1.43164 (0.04700)	1.45225 (0.04461)	1.46915 (0.0432)
			1.4	1.46602 (0.15194)	1.45609 **(0.0426)**	1.43919 (0.04447)	1.41948 (0.04725)	1.43955 (0.04443)	1.45586 (0.0427)
100	0.5	0.5	0.4	0.51758 (0.00838)	0.51341 **(0.0028**)	0.51702 (0.00297)	0.52012 (0.00308)	0.51804 (0.00300)	0.51911 (0.0030)
			1.4	0.51758 (0.00838)	0.51591 **(0.0029)**	0.51950 (0.00306)	0.52259 (0.00319)	0.52051 (0.00309)	0.52158 (0.0031)
		1.0	0.4	1.03515 (0.03295)	1.02469 **(0.0112)**	1.03191 (0.01169)	1.03809 (0.01212)	1.03288 (0.01175)	1.03715 (0.0120)
			1.4	1.03515 (0.03295)	1.02447 **(0.0111)**	1.03158 (0.01155)	1.03771 (0.01198)	1.03254 (0.01161)	1.03677 (0.0119)
		1.5	0.4	1.55273 (0.07305)	1.53388 (**0.0250**)	1.54468 (0.02591)	1.55394 (0.02682)	1.54455 (0.02589)	1.55412 (0.0268)
			1.4	1.55273 (0.07305)	1.52577 **(0.0240)**	1.53641 (0.02475)	1.54553 (0.02549)	1.53630 (0.02474)	1.54567 (0.0255)

From table 6.1 and table 6.2 we conclude that Al-Bayyati's loss function gives the minimum MSE as compared to the other loss functions and among the priors Gamma prior gives the less MSE than other assumed priors.

Conclusion

In this chapter, we have paralleled the Baye's estimates of the parameter of the Exponential-Rayleigh (ER) distribution under extension of Jeffrey's prior and gamma prior using different loss functions with that of maximum likelihood estimate. From the results, Al-Bayyati's loss function gives the minimum MSE as compared to the other loss functions and among the priors Gamma prior gives the less MSE than other assumed priors.

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