Chapter 2:

Parameter Estimation of Weighted New Weibull Pareto Distribution

Sofi Mudasir and S.P Ahmad

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Additional information is available at the end of the chapter

Introduction

Weighted distributions occur commonly in studies related to reliability, survival analysis, biomedicine, ecology, analysis of family data, and several other areas. There are number of authors worked on weighted distributions among them are Monsef and Ghoneim (2015) proposed weighted Kumaraswamy distribution for modeling some biological data, Sofi Mudasir and Ahmad (2015) study the length biased Nakagami distribution, Jan et al. (2017) studied the weighted Ailamujia distribution and find its applications to real data sets, Sofi Mudasir and Ahmad (2017) estimate the scale parameter of weighted Erlang distribution through classical and Bayesian methods of estimation, Dar et al. studied the characterization and estimation of Weighted Maxwell distribution (2018).

If $V \geq 0$ is a random variable with density function $f(v)$ and $w(v,\theta) \geq 0$ is a weight function, then the weighted random variable V_W has the probability density function given by

$$f_w(v) = Zw(v,\theta) f(v) \tag{2.1}$$

Where Z is the normalizing constant.

When $w(v,\theta) = v^{\theta}, \theta > 0,$ then the distribution is called the weighted distribution of order θ. The probability density function of WNWP distribution is obtained by using (2.1) and is given by

$$f_w(v) = \frac{\beta \eta^{\frac{\theta}{\beta}+1}}{\alpha^{\beta+\theta}\Gamma\left(\frac{\theta}{\beta}+1\right)} v^{\beta+\theta-1} \exp\left(-\eta\left(\frac{v}{\alpha}\right)^{\beta}\right), v \geq 0; \alpha, \beta, \eta, \theta > 0. \tag{2.2}$$

The corresponding cumulative distribution function is

$$F(v)=\frac{1}{\Gamma\left(\frac{\theta}{\beta}+1\right)}\gamma\left(\frac{\theta}{\beta}+1,\eta\left(\frac{v}{\alpha}\right)^{\beta}\right) \tag{2.3}$$

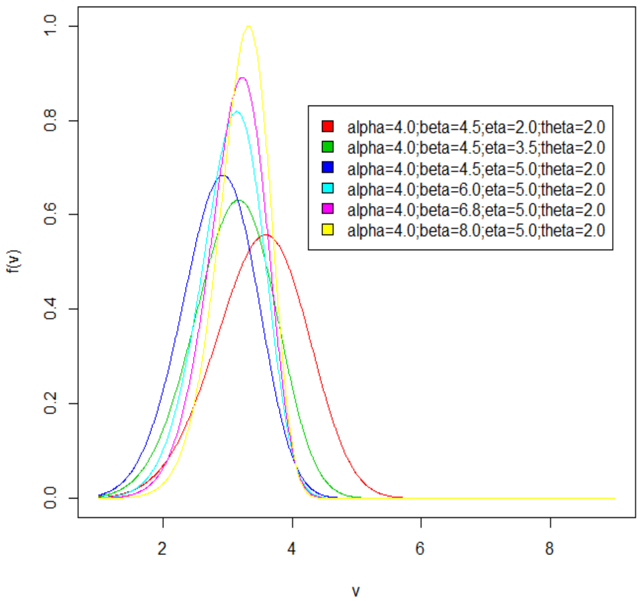

Figure 2.1: *Graph of Probability density function of Weighted Weibull Pareto distribution with different values of parameters*

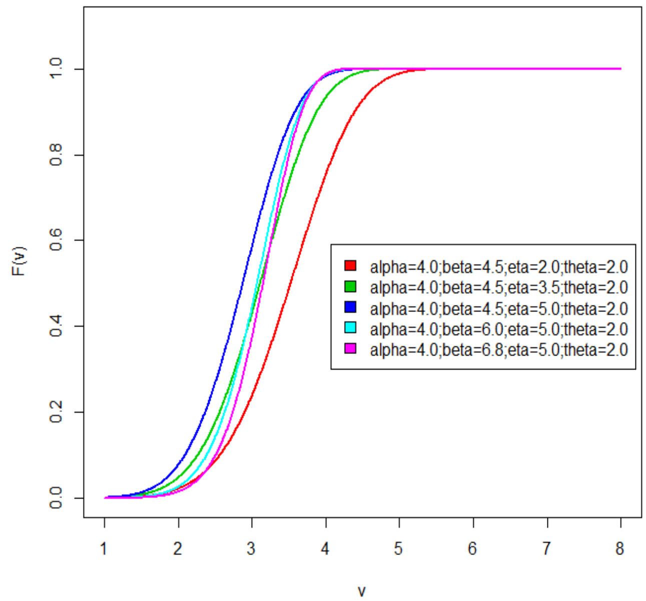

Figure 2.2: *Graph of Cumulative distribution function of Weibull Pareto distribution with different values of parameters*

Estimation Procedures

This section is devoted to three parameter estimation procedures: method of moments (MOM), maximum likelihood method of estimation (MLE), and Bayesian method of estimation.

Method of Moments (MOM)

Method of moments is a popular technique for parameter estimation. The moment estimator for the scale parameter α can be obtained by equating the first sample moment to the corresponding population moment and is given by

$$\hat{\alpha}=\frac{\eta^{\frac{1}{\beta}}\rho_{\theta}}{\rho_{\theta+1}}\bar{v}.$$

where $\rho_{\theta+s} = \Gamma\left(\frac{\theta+s}{\beta}+1\right)$.

Method of Maximum Likelihood Estimation (MLE)

Let $v_1, v_2, ..., v_n$ be a random sample from the WNWP distribution with parameter vector $\Theta = (\alpha, \beta, \eta, \theta)$. By considering (1), the likelihood function is given by

$$L(\Theta) = \left(\frac{\beta\eta^{\frac{\theta}{\beta}+1}}{\alpha^{\beta+\theta}\Gamma\left(\frac{\theta}{\beta}+1\right)}\right)^n \left(\prod_{i=1}^{n} v_i^{\beta+\theta-1}\right) \exp\left(-\frac{\eta}{\alpha^\beta}t\right).$$

The log-likelihood function can be expressed as

$$l(\Theta) = n\log\left(\frac{\beta\eta^{\frac{\theta}{\beta}+1}}{\alpha^{\beta+\theta}\Gamma\left(\frac{\theta}{\beta}+1\right)}\right) + (\beta+\theta-1)\sum_{i=1}^{n}\log(v_i) - \frac{\eta}{\alpha^\beta}t. \quad (2.4)$$

In order to estimate α , differentiate eq.(4) w.r.t. α and equate to zero, we get

$$\hat{\alpha} = \left(\frac{\beta\eta t}{n(\beta+\theta)}\right)^{\frac{1}{\beta}}.$$

Where $t = \sum_{i=1}^{n} v_i^{\beta}$.

Bayesian Method of Estimation

Here we try to find Bayes estimator for the scale parameter α for the pdf defined in (2.2). We use different priors and different loss functions.

Posterior Distribution Under the Assumption of Extension of Jeffrey's Prior

The extension of Jeffrey's prior relating scale parameter α is given as

$$\pi_1(\alpha) \propto \frac{1}{\alpha^{2c_1}}, \alpha > 0, c_1 \in R^+$$

Remark 1:

If $c_1 = 0,$ we get uniform prior, i.e.,

$\pi_{11}(\alpha) = q$, where q is constant of proportionality.

Remark 2:

If $c_1 = \frac{1}{2},$ we have $\pi_{12}(\alpha) \propto \frac{1}{\alpha}$ which is Jeffrey's prior.

Remark 3:

If $c_1 = \frac{3}{2},$ we get Hartigan's prior, i.e.,

$$\pi_{13}(\alpha) \propto \frac{1}{\alpha^3}.$$

The posterior distribution of scale parameter α under extension of Jeffrey's prior is given as

$$P_1(\alpha \mid \underline{v}) = \frac{\beta(\eta t)^{\frac{n\theta+2c_1-1}{\beta}+n} \exp\left(-\frac{\eta}{\alpha^\beta} t\right)}{\Gamma\left(\frac{n\theta+2c_1-1}{\beta}+n\right)\alpha^{n\theta+n\beta+2c_1}}. \qquad (2.5)$$

Posterior Distribution Under the Assumption of Quasi Prior

The quasi prior relating to the scale parameter α is given as

$$\pi_2(\alpha) \propto \frac{1}{\alpha^{d_1}}, \alpha > 0, d_1 > 0.$$

The posterior distribution under quasi prior is given as

$$P_2(\alpha \mid \underline{v}) = \frac{\beta(\eta t)^{\frac{n\theta+d_1-1}{\beta}+n} \exp\left(-\frac{\eta}{\alpha^\beta} t\right)}{\Gamma\left(\frac{n\theta+d_1-1}{\beta}+n\right)\alpha^{n\theta+n\beta+d_1}}. \qquad (2.6)$$

Bayes Estimator Under Squared Error Loss Function (SELF) Using Extension of Jeffrey's Prior

The SELF relating to the parameter α is defined as

$$L(\hat{\alpha}-\alpha) = b(\hat{\alpha}-\alpha)^2$$

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Where b is a constant and $\hat{\alpha}$ is the estimator of α .

Risk function under SELF using extension of Jeffrey's prior is given by

$$R(\hat{\alpha})=\int_0^\infty b(\hat{\alpha}-\alpha)^2 P_1(\alpha\mid \underline{v})d\alpha .$$

$$=b\hat{\alpha}^2+b(\eta t)^{\frac{2}{\beta}}\frac{\Gamma\left(\frac{n\theta+2c_1-3}{\beta}+n\right)}{\Gamma\left(\frac{n\theta+2c_1-1}{\beta}+n\right)}-2b(\eta t)^{\frac{1}{\beta}}\frac{\Gamma\left(\frac{n\theta+2c_1-2}{\beta}+n\right)}{\Gamma\left(\frac{n\theta+2c_1-1}{\beta}+n\right)}\hat{\alpha} .$$

Minimization of risk function w.r.t. $\hat{\alpha}$ gives us the Bayes estimator as

$$\hat{\alpha}=\frac{\Gamma\left(\frac{n\theta+2c_1-2}{\beta}+n\right)}{\Gamma\left(\frac{n\theta+2c_1-1}{\beta}+n\right)}(\eta t)^{\frac{1}{\beta}} .$$

Bayes Estimator Under the Combination of Quadratic Loss Function (QLF) And Extension of Jeffrey's Prior

Risk function under QLF using extension of Jeffrey's prior is given by

$$R(\hat{\alpha})=\int_0^\infty\left(\frac{\hat{\alpha}-\alpha}{\alpha}\right)^2 P_1(\alpha\mid \underline{v})\,d\alpha$$

$$=\frac{\Gamma\left(\frac{n\theta+2c_1+1}{\beta}+n\right)}{\Gamma\left(\frac{n\theta+2c_1-1}{\beta}+n\right)(\eta t)^{\frac{2}{\beta}}}\hat{\alpha}^2+1-2\frac{\Gamma\left(\frac{n\theta+2c_1}{\beta}+n\right)}{\Gamma\left(\frac{n\theta+2c_1-1}{\beta}+n\right)(\eta t)^{\frac{1}{\beta}}}\hat{\alpha} .$$

Now the solution of $\frac{\partial\left(R(\hat{\alpha})\right)}{\partial\hat{\alpha}}=0$ is the required Bayes estimator and is given by

$$\hat{\alpha}=\frac{\Gamma\left(\frac{n\theta+2c_1}{\beta}+n\right)}{\Gamma\left(\frac{n\theta+2c_1+1}{\beta}+n\right)}(\eta t)^{\frac{1}{\beta}}.$$

Bayes Estimator Under the Combination of Al-Bayyati's Loss Function (ALF) and Extension of Jeffrey's Prior

The risk function under the combination of ALF and extension of Jeffrey's prior is

$$R(\hat{\alpha})=\int_0^\infty \alpha^{c_2}(\hat{\alpha}-\alpha)^2 P_1(\alpha\mid \underline{v})d\alpha$$

$$=\frac{\Gamma\left(\frac{n\theta+2c_1-c_2-1}{\beta}+n\right)}{\Gamma\left(\frac{n\theta+2c_1-1}{\beta}+n\right)}(\eta t)^{\frac{c_2}{\beta}}\hat{\alpha}^2+\frac{\Gamma\left(\frac{n\theta+2c_1-c_2-3}{\beta}+n\right)}{\Gamma\left(\frac{n\theta+2c_1-1}{\beta}+n\right)}(\eta t)^{\frac{c_2+2}{\beta}}-$$
$$2\frac{\Gamma\left(\frac{n\theta+2c_1-c_2-2}{\beta}+n\right)}{\Gamma\left(\frac{n\theta+2c_1-1}{\beta}+n\right)}(\eta t)^{\frac{c_2+1}{\beta}}\hat{\alpha}.$$

On solving $\frac{\partial\left(R(\hat{\alpha})\right)}{\partial\hat{\alpha}}=0$ for $\hat{\alpha}$, we get the Bayes estimator given as

$$\hat{\alpha}=\frac{\Gamma\left(\frac{n\theta+2c_1-c_2-2}{\beta}+n\right)}{\Gamma\left(\frac{n\theta+2c_1-c_2-1}{\beta}+n\right)}(\eta t)^{\frac{1}{\beta}}$$

Bayes Estimator Under Squared Error Loss Function (SELF) Using Quasi Prior

Under the combination of SELF and quasi prior, the risk function is given by

$$R(\hat{\alpha})=\int_0^\infty b(\hat{\alpha}-\alpha)^2 P_2(\alpha\mid \underline{v})d\alpha\,. \qquad (2.7)$$

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After substituting the value of eq. (2.6) in eq. (2.7) and simplification, we get

$$R(\hat{\alpha}) = b\hat{\alpha}^2 + b(\eta\, t)^{\frac{2}{\beta}} \frac{\Gamma\left(\frac{n\theta + d_1 - 3}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - 1}{\beta} + n\right)} - 2b(\eta\, t)^{\frac{1}{\beta}} \frac{\Gamma\left(\frac{n\theta + d_1 - 2}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - 1}{\beta} + n\right)}\hat{\alpha}.$$

The solution of $\frac{\partial(R(\hat{\alpha}))}{\partial\hat{\alpha}} = 0$ is the required Bayes estimator and is given by

$$\hat{\alpha} = \frac{\Gamma\left(\frac{n\theta + d_1 - 2}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - 1}{\beta} + n\right)}(\eta\, t)^{\frac{1}{\beta}}$$

Bayes Estimator Under Quadratic Loss Function (QLF) Using Quasi Prior

The risk function under the combination of QLF and quasi prior is given by

$$R(\hat{\alpha}) = \frac{\Gamma\left(\frac{n\theta + d_1 + 1}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - 1}{\beta} + n\right)(\eta\, t)^{\frac{2}{\beta}}}\hat{\alpha}^2 + 1 - 2\frac{\Gamma\left(\frac{n\theta + d_1}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - 1}{\beta} + n\right)(\eta\, t)^{\frac{1}{\beta}}}\hat{\alpha}$$

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Minimization of risk function w.r.t. $\hat{\alpha}$ gives us the Bayes estimator as

$$\hat{\alpha} = \frac{\Gamma\left(\frac{n\theta + d_1}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 + 1}{\beta} + n\right)}(\eta\, t)^{\frac{1}{\beta}}$$

Bayes Estimator Under the Combination of Al-Bayyati's Loss Function (ALF) and Quasi Prior

The risk function under the combination of ALF and quasi prior is given by

$$R(\hat{\alpha}) = \frac{\Gamma\left(\frac{n\theta + d_1 - c_2 - 1}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - 1}{\beta} + n\right)} (\eta t)^{\frac{c_2}{\beta}} \hat{\alpha}^2 + \frac{\Gamma\left(\frac{n\theta + d_1 - c_2 - 1}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - 1}{\beta} + n\right)} (\eta t)^{\frac{c_2}{\beta}} - 2\frac{\Gamma\left(\frac{n\theta + d_1 - c_2 - 2}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - 1}{\beta} + n\right)} (\eta t)^{\frac{c_2+1}{\beta}} \hat{\alpha}.$$

On solving $\frac{\partial(R(\hat{\alpha}))}{\partial\hat{\alpha}} = 0$ for $\hat{\alpha}$, we get the Bayes estimator given as

$$\hat{\alpha} = \frac{\Gamma\left(\frac{n\theta + d_1 - c_2 - 2}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - c_2 - 1}{\beta} + n\right)} (\eta t)^{\frac{1}{\beta}}$$

The Bayes estimates using different priors under different loss functions are given below in table 2.1.

Table 2.1: *Bayes estimators under different combinations of loss functions and prior distributions*

Prior	Loss function	Estimator
Extension of Jeffrey's prior	Squared error	$\hat{\alpha} = \frac{\Gamma\left(\frac{n\theta + 2c_1 - 2}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + 2c_1 - 1}{\beta} + n\right)} (\eta t)^{\frac{1}{\beta}}$
	Quadratic	$\hat{\alpha} = \frac{\Gamma\left(\frac{n\theta + 2c_1}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + 2c_1 + 1}{\beta} + n\right)} (\eta t)^{\frac{1}{\beta}}$
	Al-Bayyati's	$\hat{\alpha} = \frac{\Gamma\left(\frac{n\theta + 2c_1 - c_2 - 2}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + 2c_1 - c_2 - 1}{\beta} + n\right)} (\eta t)^{\frac{1}{\beta}}$

Quasi prior	Squared error	$\hat{\alpha} = \dfrac{\Gamma\left(\frac{n\theta + d_1 - 2}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - 1}{\beta} + n\right)} (\eta t)^{\frac{1}{\beta}}$
	Quadratic	$\hat{\alpha} = \dfrac{\Gamma\left(\frac{n\theta + d_1}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 + 1}{\beta} + n\right)} (\eta t)^{\frac{1}{\beta}}$
	Al-Bayyati's	$\hat{\alpha} = \dfrac{\Gamma\left(\frac{n\theta + d_1 - c_2 - 2}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + d_1 - c_2 - 1}{\beta} + n\right)} (\eta t)^{\frac{1}{\beta}}$
Hartigan's prior	Squared error	$\hat{\alpha} = \dfrac{\Gamma\left(\frac{n\theta + 1}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + 2}{\beta} + n\right)} (\eta t)^{\frac{1}{\beta}}$
	Quadratic	$\hat{\alpha} = \dfrac{\Gamma\left(\frac{n\theta + 3}{\beta} + n\right)}{\Gamma\left(\frac{n\theta + 4}{\beta} + n\right)} (\eta t)^{\frac{1}{\beta}}$
	Al-Bayyati's	$\hat{\alpha} = \dfrac{\Gamma\left(\frac{n\theta - c_2 + 1}{\beta} + n\right)}{\Gamma\left(\frac{n\theta - c_2 + 2}{\beta} + n\right)} (\eta t)^{\frac{1}{\beta}}$

Data Analysis

In this subdivision we analyze two real life data sets for illustration of the proposed procedure. The first data set represents the exceedances of flood peaks (m^3/s) of the Wheaton river near car cross in Yukon territory, Canada. The data set consists of 72 exceedances for the year 1958-1984, rounded to one decimal place. The second data set represents the survival times (in days) of guinea pigs injected with different doses of tubercle bacilli.

Data set 2.1. Exceedances of flood peaks (m^3/s) of the Wheaton River near Carcross in Yukon Territory, Canada. The data consists of 72 exceedances for the year 1958-1984, rounded to one decimal place as shown below.

1.7	2.2	14.4	1.1	0.4	20.6	5.3	0.7
1.4	18.7	8.5	25.5	11.6	14.1	22.1	1.1
0.6	2.2	39.0	0.3	15.0	11.0	7.3	22.9
0.9	1.7	7.0	20.1	0.4	2.8	14.1	9.9
5.6	30.8	13.3	4.2	25.5	3.4	11.9	21.5
1.5	2.5	27.4	1.0	27.1	20.2	16.8	5.3
1.9	10.4	13.0	10.7	12.0	30.0	9.3	3.6
2.5	27.6	14.4	36.4	1.7	2.7	37.6	64.0
1.7	9.7	0.1	27.5	1.1	2.5	0.6	27.0

Data set 2.2. The data set is from Kundu & Howlader (2010), the data set represents the survival times (in days) of guinea pigs injected with different doses of tubercle bacilli. The regimen number is the common logarithm of the number of bacillary units per 0.5 ml. (log (4.0) 6.6). Corresponding to regimen 6.6, there were 72 observations listed below:

12	15	22	24	24	32	32	33	34	38	38	43	44	48
52	53	54	54	55	56	57	58	58	59	60	60	60	60
61	62	63	65	65	67	68	70	70	72	73	75	76	76
81	83	84	85	87	91	95	96	98	99	109	110	121	127
129	131	143	146	146	175	175	211	233	258	258	263	297	341
341	376												

Table 2.2: *Estimates and (posterior risk) under extension of Jeffrey's prior using different loss functions using data set 1.*

θ	β	η	**B**	c_1	c_2	**MOM**	**MLE**	**SELF**	**QLF**	**ALF**
1.0	4.0	1.0	1.5	1.0	1.0	12.36694	169.11393	169.29035 (30.030345)	169.05506 (0.0006941971)	169.40861 (3403.479)
	4.5	1.5	2.0	1.5	1.5	13.63276	105.17297	105.21717 (12.467706)	105.09922 (0.0005599442)	105.30611 (6766.528)
	5.5	2.5	2.5	2.5	2.0	14.81888	49.80533	49.79078 (2.407005)	49.75225 (0.0003865627)	49.82950 (2401.806)
	6.0	3.0	3.0	3.0	2.5	15.08626	37.15070	37.13227 (1.365364)	37.10784 (0.0003286734)	37.16296 (3851.661)
2.0	4.0	1.0	1.5	1.0	1.0	11.65966	161.57869	161.71912 (22.814399)	161.53183 (0.0005785327)	161.81316 (2468.287)
	4.5	1.5	2.0	1.5	1.5	12.98237	101.34020	101.37625 (9.788270)	101.28007 (0.0004739584)	101.44871 (5019.687)
	5.5	2.5	2.5	2.5	2.0	14.30474	48.52619	48.51391 (1.980545)	48.48136 (0.0003352443)	48.54660 (1874.647)
	6.0	3.0	3.0	3.0	2.5	14.63197	36.33304	36.31726 (1.143087)	36.29634 (0.0002878088)	36.34353 (3047.827)
3.0	4.0	1.0	1.5	1.0	1.0	11.11300	155.47028	155.58608 (18.087358)	155.43165 (0.0004959065)	155.66358 (1881.713)
	4.5	1.5	2.0	1.5	1.5	12.46849	98.16827	98.19853 (7.956599)	98.11778 (0.0004108653)	98.25934 (3887.504)
	5.5	2.5	2.5	2.5	2.0	13.88447	47.43435	47.42376 (1.669945)	47.39567 (0.0002959546)	47.45196 (1509.457)
	6.0	3.0	3.0	3.0	2.5	14.25562	35.62675	35.61301 (0.977227)	35.59476 (0.0002559819)	35.63591 (2479.360)

Table 2.3: *Estimates and (posterior risk) under Quasi prior using different loss functions using data set 1.*

θ	β	η	**b**	c_2	d_1	**MOM**	**MLE**	**SELF**	**QLF**	**ALF**
1.0	4.0	1.0	1.0	1.0	1.0	12.36694	169.11393	169.40861 (30.156580)	169.17250 (0.0006961301)	169.52728 (262.2463)
	4.5	1.5	1.5	1.5	1.5	13.63276	105.17297	105.30611 (12.536442)	105.18761 (0.0005620686)	105.39546 (662.6073)
	5.5	2.5	2.5	2.0	2.5	14.81888	49.80533	49.83921 (2.424638)	49.80044 (0.0003886283)	49.87818 (342.8307)
	6.0	3.0	3.0	2.5	3.0	15.08626	37.15070	37.16912 (1.376253)	37.14453 (0.0003306295)	37.20003 (637.4309)
2.0	4.0	1.0	1.0	1.0	1.0	11.65966	161.57869	161.8132 (22.894200)	161.62534 (0.0005798746)	161.90749 (194.5057)
	4.5	1.5	1.5	1.5	1.5	12.98237	101.34020	101.4487 (9.833878)	101.35215 (0.0004754796)	101.52146 (500.4294)
	5.5	2.5	2.5	2.0	2.5	14.30474	48.52619	48.5548 (1.993109)	48.52207 (0.0003367968)	48.58767 (270.8228)
	6.0	3.0	3.0	2.5	3.0	14.63197	36.33304	36.3488 (1.151059)	36.32775 (0.0002893075)	36.37523 (509.4890)
3.0	4.0	1.0	1.0	1.0	1.0	11.11300	155.47028	155.66358 (18.1415295)	155.50877 (0.0004968921)	155.74127 (151.1310)
	4.5	1.5	1.5	1.5	1.5	12.46849	98.16827	98.25934 (7.9887023)	98.17831 (0.0004120079)	98.32035 (393.5805)
	5.5	2.5	2.5	2.0	2.5	13.88447	47.43435	47.45903 (1.6792879)	47.43080 (0.0002971639)	47.48736 (220.3956)
	6.0	3.0	3.0	2.5	3.0	14.25562	35.62675	35.64050 (0.9832819)	35.62215 (0.0002571668)	35.66352 (418.1959)

Table 2.4: *Estimates and (posterior risk) under Hartigan's prior using different loss functions using data set 1.*

θ	β	η	b	c_2	d_1	MOM	MLE	SELF	QLF	ALF
1.0	4.0	1.0	1.0	1.0	1.0	12.36694	169.11393	169.17250 (29.904988)	168.93803 (0.0006922748)	169.29035 (259.8740)
	4.5	1.5	1.5	1.5	1.5	13.63276	105.17297	105.21717 (12.467706)	105.09922 (0.0005599442)	105.30611 (658.4081)
	5.5	2.5	2.5	2.0	2.5	14.81888	49.80533	49.82950 (2.421093)	49.79078 (0.0003882134)	49.86842 (342.2278)
	6.0	3.0	3.0	2.5	3.0	15.08626	37.15070	37.16912 (1.376253)	37.14453 (0.0003306295)	37.20003 (637.4309)
2.0	4.0	1.0	1.0	1.0	1.0	11.65966	161.57869	161.6253 (22.735061)	161.43860 (0.0005771970)	161.71912 (193.0399)
	4.5	1.5	1.5	1.5	1.5	12.98237	101.34020	101.3762 (9.788270)	101.28007 (0.0004739584)	101.44871 (497.7475)
	5.5	2.5	2.5	2.0	2.5	14.30474	48.52619	48.5466 (1.990585)	48.51391 (0.0003364852)	48.57944 (270.4103)
	6.0	3.0	3.0	2.5	3.0	14.63197	36.33304	36.3488 (1.151059)	36.32775 (0.0002893075)	36.37523 (509.4894)
3.0	4.0	1.0	1.0	1.0	1.0	11.11300	155.47028	155.50877 (18.0334558)	155.35472 (0.0004949247)	155.58608 (150.1550)
	4.5	1.5	1.5	1.5	1.5	12.46849	98.16827	98.19853 (28.1517503)	98.11778 (0.0004108653)	98.25934 (391.7533)
	5.5	2.5	2.5	2.0	2.5	13.88447	47.43435	47.45196 (1.6774122)	47.42376 (0.0002969213)	47.48027 (220.0996)
	6.0	3.0	3.0	2.5	3.0	14.25562	35.62675	35.64050 (0.9832819)	35.62215 (0.0002571668)	35.66352 (418.1959)

Table 2.5: *Estimates and (posterior risk) under extension of Jeffrey's prior using different loss functions using data set 2.*

θ	β	η	B	c_1	c_2	MOM	MLE	SELF	QLF	ALF
1.0	4.0	1.0	1.0	1.0	1.0	102.0919	2473.4000	2475.9802 (6423.77429)	2472.5390 (0.0006941971)	2477.7098 (10647998)
	4.5	1.5	1.5	1.5	1.5	112.5415	1141.7260	1142.2058 (1469.27156)	1140.9254 (0.0005599442)	1143.1713 (28521214)
	5.5	2.5	2.5	2.5	2.0	122.3332	350.4549	350.3525 (119.17625)	350.0814 (0.0003865627)	350.6250 (5887940)
	6.0	3.0	3.0	3.0	2.5	124.5405	222.1827	222.0725 (48.83544)	221.9264 (0.0003286734)	222.2561 (12050165)
2.0	4.0	1.0	1.0	1.0	1.0	96.25317	2363.1922	2365.2461 (4880.21539)	2362.5070 (0.0005785327)	2366.6216 (7722190)
	4.5	1.5	1.5	1.5	1.5	107.17238	1100.1187	1100.5099 (1153.51019)	1099.4659 (0.0004739584)	1101.2966 (21158203)
	5.5	2.5	2.5	2.5	2.0	118.08883	341.4543	341.3678 (98.06123)	341.1388 (0.0003352443)	341.5979 (4595628)
	6.0	3.0	3.0	3.0	2.5	120.79026	217.2926	217.1983 (40.88520)	217.0732 (0.0002878088)	217.3554 (9535319)
3.0	4.0	1.0	1.0	1.0	1.0	91.74031	2273.8528	2275.5464 (3869.05665)	2273.2878 (0.0004959065)	2276.6800 (26855836)
	4.5	1.5	1.5	1.5	1.5	102.93017	1065.6851	1066.0136 (937.65483)	1065.1370 (0.0004108653)	1066.6737 (64929980)
	5.5	2.5	2.5	2.5	2.0	114.61948	333.7715	333.6970 (82.68276)	333.4994 (0.0002959546)	333.8955 (11511443)
	6.0	3.0	3.0	3.0	2.5	117.68336	213.0687	212.9864 (34.95281)	212.8773 (0.0002559819)	213.1234 (21993921)

Table 2.6: *Estimates and (posterior risk) under Quasi prior using different loss functions using data set 2.*

θ	β	η	b	c_2	d_1	MOM	MLE	SELF	QLF	ALF
1.0	4.0	1.0	1.0	1.0	1.0	102.0919	2473.4000	2477.7098 (6450.77712)	2474.2566 (0.0006961301)	2479.4455 (214534.4)
	4.5	1.5	1.5	1.5	1.5	112.5415	1141.7260	1143.1713 (1477.37186)	1141.8850 (0.0005620686)	1144.1413 (847675.2)
	5.5	2.5	2.5	2.0	2.5	122.3332	350.4549	350.6933 (120.04928)	350.4205 (0.0003886283)	350.9675 (316830.4)
	6.0	3.0	3.0	2.5	3.0	124.5405	222.1827	222.2929 (49.22489)	222.1458 (0.0003306295)	222.4778 (815467.2)
2.0	4.0	1.0	1.0	1.0	1.0	96.25317	2363.1922	2366.6216 (4897.28552)	2363.8745 (0.0005798746)	2368.0012 (159118.2)
	4.5	1.5	1.5	1.5	1.5	107.17238	1100.1187	1101.2966 (1158.88497)	1100.2484 (0.0004754796)	1102.0863 (640200.6)
	5.5	2.5	2.5	2.0	2.5	118.08883	341.4543	341.6556 (98.68333)	341.4253 (0.0003367968)	341.8869 (250283.6)
	6.0	3.0	3.0	2.5	3.0	120.79026	217.2926	217.3869 (41.17032)	217.2611 (0.0002893075)	217.5450 (651791.2)
3.0	4.0	1.0	1.0	1.0	1.0	91.74031	2273.8528	2276.6800 (3880.64442)	2274.4157 (0.0004968921)	2277.8163 (123634.9)
	4.5	1.5	1.5	1.5	1.5	102.93017	1065.6851	1066.6737 (941.43805)	1065.7941 (0.0004120079)	1067.3360 (503508.5)
	5.5	2.5	2.5	2.0	2.5	114.61948	333.7715	333.9452 (83.14533)	333.7466 (0.0002971639)	334.1446 (203680.8)
	6.0	3.0	3.0	2.5	3.0	117.68336	213.0687	213.1509 (35.16938)	213.0411 (0.0002571668)	213.2885 (534999.0)

Table 2.7: *Estimates and (posterior risk) under Hartigan's prior using different loss functions using data set 2.*

θ	β	η	b	c_2	d_1	MOM	MLE	SELF	QLF	ALF
1.0	4.0	1.0	1.0	1.0	1.0	102.0919	2473.4000	2474.2566 (6396.95932)	2470.8273 (0.0006922748)	2475.9802 (212593.6)
	4.5	1.5	1.5	1.5	1.5	112.5415	1141.7260	1142.2058 (1469.27156)	1140.9254 (0.0005599442)	1143.1713 (842303.2)
	5.5	2.5	2.5	2.0	2.5	122.3332	350.4549	350.6250 (119.87379)	350.3525 (0.0003882134)	350.8989 (316273.3)
	6.0	3.0	3.0	2.5	3.0	124.5405	222.1827	222.2929 (49.22489)	222.1458 (0.0003306295)	222.4778 (815467.2)
2.0	4.0	1.0	1.0	1.0	1.0	96.25317	2363.1922	2363.8745 (4863.24419)	2361.1433 (0.0005771970)	2365.246 (157919.1)
	4.5	1.5	1.5	1.5	1.5	107.17238	1100.1187	1100.5099 (1153.51019)	1099.4659 (0.0004739584)	1101.297 (636769.9)
	5.5	2.5	2.5	2.0	2.5	118.08883	341.4543	341.5979 (98.55836)	341.3678 (0.0003364852)	341.829 (249902.5)
	6.0	3.0	3.0	2.5	3.0	120.79026	217.2926	217.3869 (41.17032)	217.2611 (0.0002893075)	217.5450 (651791.2)
3.0	4.0	1.0	1.0	1.0	1.0	91.74031	2273.8528	2274.4157 (3857.52644)	2272.1627 (0.0004949247)	2275.5464 (122836.5)
	4.5	1.5	1.5	1.5	1.5	102.93017	1065.6851	1066.0136 (937.65483)	1065.1370 (0.0004108653)	1066.6737 (501170.9)
	5.5	2.5	2.5	2.0	2.5	114.61948	333.7715	333.8955 (83.05246)	333.6970 (0.0002969213)	334.0946 (203407.2)
	6.0	3.0	3.0	2.5	3.0	117.68336	213.0687	213.1509 (35.16938)	213.0411 (0.0002571668)	213.2885 (534999.0)

Conclusion

In this chapter, method of moments, maximum likelihood and Bayesian methods of estimation were studied for estimating the scale parameter of the WNWP distribution. Bayes estimators are obtained using different loss functions under different types of priors. For comparison of different loss functions and different types of priors, two real life data sets are used, and the outcomes are obtained through R-software. On equating the posterior risk obtained under different loss functions, it is clear from the above tables that QLF has minimum value of posterior risk and is thus preferable as compared to other loss functions used in this paper. It is also observed that as we increase the value of weighted parameter θ, the posterior risk decreases. Also, from tables 2.2 to 2.7, it is clear that in order to estimate the said parameter combination of quadratic loss function and extension of Jeffrey's prior can be preferred.

Author's Detail

Sofi Mudasir* and S.P Ahmad

Department of Statistics, University of Kashmir, Srinagar, India

*Corresponding author email: sofimudasir3806@gmail.com

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References

Fisher, R.A. (1934). The effects of methods of ascertainment upon the estimation of frequencies. *The Annals of Eugenics* 6, 13–25.

C.R. Rao, On discrete distributions arising out of methods of ascertainment in classical and contagious discrete distributions. Pergamon press and statistical publishing society, Calcutta, 320-332 (1965).

Monsef, M.M.E. and Ghoneim, S.A.E. (2015). The weighted kumaraswamy distribution. *International information institute, 18, 3289-3300*.

Sofi Mudasir and Ahmad, S.P. (2017). Parameter estimation of weighted Erlang distribution using R-software. *Mathematical theory and Modelling, 7, 1-21*.

Uzma Jan, Kawsar Fatima and Ahmad, S.P. (2017) on weighted Ailamujia distribution and its applications to life time data journal of statistics applications and probability, 6(3), 619-633.

Sofi Mudasir and Ahmad, S.P.(2015). Structural properties of length biased Nakagami distribution. *International Journal of Modern Mathematical Sciences*, 13(3), 217-227.

Aijaz Ahmad Dar, A. Ahmed and J.A. Reshi (2018). Characterization and estimation of weighted Maxwell distribution. *An international journal of applied mathematics and information sciences*. 12(1), 193-202.