Chapter 1:

Bayesian Analysis of Zero-Inflated Generalized Power Series Distributions Under Different Loss Functions

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Introduction

As we know some of the family members of generalized power series distributions (GPSD) like binomial, negative binomial, Poisson and logarithmic series distributions are widely used for modelling count data. The properties of modality and divisibility of these distributions are known in the literature. Misra et.al (2003), Alamatsaz and Abbasi (2008), Aghababaei Jazi and Alamatsaz (2010), Abbasi et.al (2010) and Aghababaei Jazi et.al (2010) studied the stochastic ordering comparison between these distributions and their mixtures.

For modelling count data like accumulated claims in insurance and correlated count data which exhibit over-dispersion has resulted in introduction of zero-inflated and non-zero inflated parameter counterparts of the GPS distributions. Neyman (1939) and Feller (1943) studied that in some discrete data, the observed frequency for $X = 0$ is much higher than the expected frequency predicted by the assumed model. To be more specific, let us suppose that there are two machines. One of which is perfect and does not produce any defective item. The other machine produces defective items according to a Poisson distribution. We record the joint output of the two machines without knowing whether a specific item is produced by one or the other. In this case, the zero count seems to be inflated. Pandey (1964-65) studied a situation dealing with the number of flowers of plants of Primula veris. He has found that most of the plants were with eight flowers and inflated Poisson distribution (inflated at the point 8 not zero) proved to be the best model for fitting of such a data set. A similar data set on premature ventricular contractions where the distribution turns out to be inflated binomial has been analyzed by Farewell and Sprott (1988). Yip (1988) while dealing with the number of insects per leaf came to the conclusion that inflated Poisson distribution is the best fitted model for such a data set.

Martine, et al. (2005) and Kuhnert, et al. (2005) discussed the applications of zero-inflated modeling in ecology. Kolev, et al. (2000) studied the application of inflated-parameter family of generalized power series distributions in analysis of overdispersed insurance data. Patil and Shirke (2007) and Patil and Shirke (2011a, b) also studied different aspects of the zero-inflated power series distributions. From the literature it appears that majority of the study is restricted to properties and applications of inflated generalized power series distributions and relatively less work has been done on the estimation part particularly the Bayesian estimation of inflated generalized power series distributions. We also refer the readers to Winkelmann (2000), Hassan and Ahmad (2006), and Aghababaei Jazi and Alamatsaz (2011).

In this note, we studied the Bayesian analysis of zero-inflated power series distributions under different loss function i.e. squared error loss function and weighted squared error loss function. The results obtained for the zero-inflated power series distribution are then applied to its particular cases like zero-inflated Poisson distribution and zero-inflated negative binomial distribution.

Rodrigues (2003) studied zero-inflated Poisson distribution from the Bayesian perspective using data augmentation algorithm. Gosh, et al. (2006) introduced a flexible class of zero-inflated models which includes zero-inflated Poisson (ZIP) model, as special case and developed a Bayesian estimation method as an alternative to traditionally used maximum likelihood-based methods to analyze such data. As disused above, our aim is to give Bayes estimators of functions of parameters under different loss functions of zero-inflated generalized power series distribution (ZIGPSD) represented by the following probability mass function

$$P[X=x]=\begin{cases}\alpha+(1-\alpha)\frac{a(0)}{f(\theta)}, & x=0\\ (1-\alpha)\frac{a(x)\theta^x}{f(\theta)}, & x=1,2,3,\ldots\end{cases} \qquad (1.1)$$

where $0<\alpha\leq 1$ is the probability of inflation, $f(\theta)=\sum_x a(x)\theta^x$ is a function of parameter θ and is positive, finite and differentiable and coefficients $a(x)$ are non-negative and free of θ. It is clear that for $\alpha=0$, the model (1.1) reduces to simple generalized power series distribution introduced by Patil (1961).

The whole article is divided in to different sections. Section 2 deals with the Bayes estimators of functions of parameters of zero-inflated generalized power series distribution (ZIGPSD) under squared error loss function and weighted square error loss function. Using different prior distributions and the results of zero-inflated GPSD, the Bayes estimators of functions of parameters of zero-inflated Poisson and zero-inflated negative binomial distributions are obtained in Sections 3 and 4 respectively. Finally, in Section 5, a numerical example is provided to illustrate the results and a goodness of fit test is done using the Bayes estimators.

Bayesian Estimation of Zero-Inflated GPSD

Let $X_1, X_2, \cdots, X_N$ be a random sample of size N drawn from the zero-inflated GPSD (1.1), then the likelihood function of $X_1, X_2, \cdots, X_N$ is given by

$$L(\theta, \alpha/\underline{x}) = \sum_{j=0}^{N_0} \binom{N_0}{j} \alpha^j (1-\alpha)^{N-j} (a(0))^{N_0-j} \prod_{i=1}^{N-N_0} a(x_i)\theta^t [f(\theta)]^{j-N} \tag{1.2}$$

where $\underline{x} = (x_1, x_2, \cdots, x_N)$, $t = \sum_{i=1}^{N-N_0} x_i$ and N_i is the number of observations in the i'th class such that.$\sum_{i\geq 1} N_i = N$.

For the Bayesian set up, we assumed that, priori, θ and α are independent, since in the zero-inflated distribution, an arbitrary probability is assigned to the zero class. As the parameter α represents the proportion of 'excess zeros', we may take Beta (u, v) prior as a conjugate prior for α ,with prior density function

$$g(\alpha) = \frac{\alpha^{u-1}(1-\alpha)^{v-1}}{B(u,v)}, 0 < \alpha < 1, u, v > 0 \tag{1.3}$$

where, $B(u, v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}$.

The prior distribution for θ is taken to be conjugate or non-conjugate prior distribution denoted by $h(\theta)$.

The Joint posterior probability density function (p.d.f) of θ and α corresponding to the prior $h(\theta)$ and $g(\alpha)$ respectively is given by

$$\Pi(\theta, \alpha/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}\alpha^{j+u-1}(1-\alpha)^{N-j+v-1}(a(0))^{N_0-j}\theta^t[f(\theta)]^{j-N}h(\theta)}{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta \theta^t[f(\theta)]^{j-N}h(\theta)d\theta} \tag{1.4}$$

The marginal posterior probability density functions of θ and α are respectively given by

$$\Pi(\theta/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}\alpha^{j+u-1}(1-\alpha)^{N-j+v-1}(a(0))^{N_0-j}\theta^t[f(\theta)]^{j-N}h(\theta)}{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta \theta^t[f(\theta)]^{j-N}h(\theta)d\theta} \tag{1.5}$$

$$\Pi(\alpha/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}\alpha^{j+u-1}(1-\alpha)^{N-j+v-1}(a(0))^{N_0-j}\int_\Theta \theta^t[f(\theta)]^{j-N}h(\theta)d\theta}{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta \theta^t[f(\theta)]^{j-N}h(\theta)d\theta} \tag{1.6}$$

The Bayes estimates $\hat{\eta}(\theta)$ of $\eta(\theta)$ and $\hat{\gamma}(\alpha)$ of $\gamma(\alpha)$ under the squared error loss function (SELF), where $\eta(\theta)$ and $\gamma(\alpha)$ are respectively the functions of θ and α are given by

$$\hat{\eta}_B = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta \eta(\theta)\theta^t[f(\theta)]^{j-N}h(\theta)d\theta}{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta \theta^t[f(\theta)]^{j-N}h(\theta)d\theta} \tag{1.7}$$

$$\hat{\gamma}_B = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}(a(0))^{N_0-j}\int_0^1\int_\Theta \gamma(\alpha)\alpha^{j+u-1}(1-\alpha)^{N-j+v-1}\theta^t[f(\theta)]^{j-N}h(\theta)d\theta d\alpha}{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta \theta^t[f(\theta)]^{j-N}h(\theta)d\theta} \quad (1.8)$$

Similarly, under the weighted squared error loss function (WSELF) given by $L(\eta(\theta),d)=w(\theta)(\eta(\theta)-d)^2$ and $L(\gamma(\theta),d)=z(\alpha)(\gamma(\alpha)-d)^2$, where $w(\theta)$ is a function of θ, and $z(\alpha)$ is a function of α , d is a decision, the Bayes estimate $\hat{\eta}_w$ of $\eta(\theta)$ and $\hat{\gamma}_w$ of $\gamma(\alpha)$ are given by

$$\hat{\eta}_w = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta w(\theta)\eta(\theta)\theta^t[f(\theta)]^{j-N}h(\theta)d\theta}{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta w(\theta)\theta^t[f(\theta)]^{j-N}h(\theta)d\theta} \quad (1.9)$$

$$\hat{\gamma}_w = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}(a(0))^{N_0-j}\int_0^1\int_\Theta z(\alpha)\gamma(\alpha)\alpha^{j+u-1}(1-\alpha)^{N-j+v-1}\theta^t[f(\theta)]^{j-N}h(\theta)d\theta d\alpha}{\sum_{j=0}^{N_0}\binom{N_0}{j}(a(0))^{N_0-j}\int_0^1\int_\Theta z(\alpha)\alpha^{j+u-1}(1-\alpha)^{N-j+v-1}\theta^t[f(\theta)]^{j-N}h(\theta)d\theta d\alpha} \quad (1.10)$$

Two different forms of $w(\theta)$ and $z(\alpha)$ as weights has been considered and are given below:

(i) Let $w(\theta)=\theta^{-2}, z(\alpha)=\alpha^{-2}$, The Bayes estimate $\hat{\eta}_M$ of $\eta(\theta)$ and $\hat{\gamma}_M$ of $\gamma(\alpha)$ known as the minimum expected loss (MEL) estimate are given by

$$\hat{\eta}_M = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta \eta(\theta)\theta^{t-2}[f(\theta)]^{j-N}h(\theta)d\theta}{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta \theta^{t-2}[f(\theta)]^{j-N}h(\theta)d\theta} \quad (1.11)$$

$$\hat{\gamma}_M = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}(a(0))^{N_0-j}\int_0^1\int_\Theta \gamma(\alpha)\alpha^{(j+u-2)-1}(1-\alpha)^{N-j+v-1}\theta^t[f(\theta)]^{j-N}h(\theta)d\theta d\alpha}{\sum_{j=0}^{N_0}\binom{N_0}{j}(a(0))^{N_0-j}B(j+u-2,N-j+v)\int_\Theta \theta^t[f(\theta)]^{j-N}h(\theta)d\theta} \quad (1.12)$$

(ii) Let $w(\theta)=\theta^{-2}e^{-\delta\theta}; \delta>0$ and $z(\alpha)=\alpha^{-2}e^{-\lambda\alpha}; \lambda>0$.The Bayes estimate $\hat{\eta}_E$ of $\eta(\theta)$ and $\hat{\gamma}_E$ of $\gamma(\alpha)$ known as the exponentially weighted minimum expected loss (EWMEL) estimate are given by

$$\hat{\eta}_E = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta \eta(\theta)\theta^{t-2}e^{-\delta\theta}[f(\theta)]^{j-N}h(\theta)d\theta}{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)(a(0))^{N_0-j}\int_\Theta \theta^{t-2}e^{-\delta\theta}[f(\theta)]^{j-N}h(\theta)d\theta} \quad (1.13)$$

$$\hat{\gamma}_E = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}(a(0))^{N_0-j}\int_0^1\int_\Theta \gamma(\alpha)e^{-\lambda\alpha}\alpha^{(j+u-2)-1}(1-\alpha)^{N-j+v-1}\theta^t[f(\theta)]^{j-N}h(\theta)d\theta d\alpha}{\sum_{j=0}^{N_0}\binom{N_0}{j}(a(0))^{N_0-j}B(j+u-2,N-j+v)M(j+u-2,N+u+v-2,-\lambda)\int_\Theta \theta^t[f(\theta)]^{j-N}h(\theta)d\theta} \quad (1.14)$$

where $M(a,b;z)$ is the confluent hypergeometric function and has a series representation given by

$$M(a,b;z)=\sum_{j=0}^{\infty}\frac{(a)_j z^j}{(b)_j j!} \text{ where } (a)_0=1 \quad (1.15)$$

and $(a)_j = a(a+1)(a+2)\dots\dots\dots(a+j-1)$ (1.16)

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Now, we apply the above results to zero-inflated Poisson and zero-inflated negative binomial distributions which are the special cases of the p.m.f. (1.1) and obtain the corresponding Bayes estimators of parameters in each case.

Bayesian Estimation of Zero-Inflated Poisson Distribution

A discrete random variable X is said to follow zero-inflated Poisson distribution (NZIPD) if its probability mass function is given by

$$P[X=x]=\begin{cases}\alpha+(1-\alpha)\frac{e^{-\theta}}{x!}, & x=0\\ (1-\alpha)\frac{e^{-\theta}\theta^{x}}{x!}, & x=1,2,3,\ldots\end{cases} \quad (1.17)$$

where $\theta > 0, o < \alpha < 1.$

If $\alpha = 0$, the model (1.17) reduces to classical Poisson distribution.

It is a special case of (1.1) with

$$f(\theta) = e^{\theta},\ a(x) = \frac{1}{x!}$$

In this case, the likelihood function $L(\theta, \alpha/\underline{x})$ is of the form

$$L(\theta,\alpha/\underline{x}) = \sum_{j=0}^{N_0}\binom{N_0}{j}\alpha^{j}(1-\alpha)^{N-j}\theta^{t}e^{-\theta(N-j)} \quad (1.18)$$

With gamma prior for θ given by

$$h(\theta) = \frac{a^{b}}{\Gamma b}e^{-a\theta}\theta^{b-1}, \theta, a, b > 0 \quad (1.19)$$

and beta prior for α given by (1.3), the joint Posterior probability density function of θ and α is given by

$$\Pi(\theta,\alpha/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}\alpha^{(j+u)-1}(1-\alpha)^{(N-j+v)-1}\theta^{(t+b)-1}e^{-\theta(N-j+a)}}{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)\frac{\Gamma(t+b)}{(N-j+a)^{t+b}}} \quad (1.20)$$

The marginal posterior distribution of θ and α are respectively given by

$$\Pi(\theta/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)\theta^{(t+b)-1}e^{-\theta(N-j+a)}}{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)\frac{\Gamma(t+b)}{(N-j+a)^{t+b}}} \quad (1.21)$$

$$\Pi(\alpha/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}\frac{1}{(N-j+a)^{t+b}}\,\alpha^{(j+u)-1}\,(1-\alpha)^{(N-j+v)-1}}{\sum_{j=0}^{N_0}\binom{N_0}{j}\,B(j+u,N-j+v)\frac{1}{(N-j+a)^{t+b}}} \tag{1.22}$$

Under SELF, the Bayes estimate $\hat{\theta}_B^r$ of θ^r and $\hat{\alpha}_B^r$ of α^r are given by

$$\hat{\theta}_B^r = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)\frac{\Gamma(t+b+r)}{(N-j+a)^{t+b+r}}}{\sum_{j=0}^{N_0}\binom{N_0}{j}\,B(j+u,N-j+v)\,\frac{\Gamma(t+b)}{(N-j+a)^{t+b}}} \tag{1.23}$$

$$\hat{\alpha}_B^r = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u+r,N-j+v)\frac{1}{(N-j+a)^{t+b}}}{\sum_{j=0}^{N_0}\binom{N_0}{j}\,B(j+u,N-j+v)\,\frac{1}{(N-j+a)^{t+b}}} \tag{1.24}$$

Similarly, under WSELF, when $w(\theta) = \theta^{-2}, z(\alpha) = \alpha^{-2}$, the minimum expected loss (MEL) estimate of $\eta(\theta) = \theta^r$ and $\gamma(\alpha) = \alpha^r$ are obtained as

$$\hat{\theta}_M^r = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)\frac{\Gamma(t+b-2+r)}{(N-j+a)^{t+b-2+r}}}{\sum_{j=0}^{N_0}\binom{N_0}{j}\,B(j+u,N-j+v)\,\frac{\Gamma(t+b-2)}{(N-j+a)^{t+b-2}}} \tag{1.25}$$

$$\hat{\alpha}_M^r = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u+r-2,N-j+v)\frac{1}{(N-j+a)^{t+b}}}{\sum_{j=0}^{N_0}\binom{N_0}{j}\,B(j+u-2,N-j+v)\frac{1}{(N-j+a)^{t+b}}} \tag{1.26}$$

Finally, under the weighted squared error loss function, when $w(\theta) = \theta^{-2}e^{-\delta\theta}; \delta > 0$ and $z(\alpha) = \alpha^{-2}e^{-\lambda\alpha}, \lambda > 0$, the EWMEL estimate $\eta(\theta)$ and $\gamma(\alpha)$ are given by

$$\hat{\theta}_E^r = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)\frac{\Gamma(t+b-2+r)}{(N-j+a+\delta)^{t+b-2+r}}}{\sum_{j=0}^{N_0}\binom{N_0}{j}\,B(j+u,N-j+v)\frac{\Gamma(t+b-2)}{(N-j+a+\delta)^{t+b-2}}} \tag{1.27}$$

$$\hat{\alpha}_E^r = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u+r-2,N-j+v)M(J+u+r-2,N+u+v+r-2;\lambda)\frac{1}{(N-j+a)^{t+b}}}{\sum_{j=0}^{N_0}\binom{N_0}{j}\,B(j+u-2,N-j+v)M(J+u-2,N+u+v-2\,;\lambda)\frac{1}{(N-j+a)^{t+b}}} \tag{1.28}$$

Bayesian Estimation of Zero Inflated Negative Binomial Distribution

A discrete random variable X is said to have zero-inflated negative binomial distribution (ZINBD) if its probability mass function is given by

$$P[X = x] = \begin{cases} \alpha + (1-\alpha)(1-\theta)^m, & x = 0 \\ (1-\alpha)\binom{m+x-1}{x}\theta^x(1-\theta)^m, & x = 1, 2, 3, \dots \dots . \end{cases} \tag{1.29}$$

where $0 < \theta < 1, 0 < \alpha \leq 1$

It is a special case of (1.1) with $f(\theta) = (1-\theta)^{-m}$ and $a(x) = \binom{m+x-1}{x}$.

If $\alpha = 0$, the model (1.29) reduces to binomial distribution.

In this case the likelihood function $L(\theta, \alpha/\underline{x}\,)$ is given by

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$$L(\theta, \alpha/\underline{x}) \propto \sum_{j=0}^{N_0} \binom{N_0}{j} \alpha^j (1-\alpha)^{N-j} \theta^t (1-\theta)^{mN-mj} \quad (1.30)$$

Since $0 < \theta < 1$, we have taken two different prior distributions for θ given below

$$h_1(\theta) = \frac{\theta^{a-1}(1-\theta)^{b-1}}{B(a,b)}, 0 < \theta < 1, a, b > 0 \quad (1.31)$$

where $B(a,b) = \frac{\Gamma a \Gamma b}{\Gamma(a+b)}$ and

$$h_2(\theta) = \frac{e^{-c\theta}\theta^{a-1}(1-\theta)^{b-1}}{B(a,b)M(a,a+b;-c)}, 0 < \theta < 1, a, b > 0, \quad (1.32)$$

where $M(a, b; z)$ is the confluent hypergeometric function and has a series representation given by (1.15) and (1.16)

The joint posterior p.d.f of θ and α corresponding to the prior $h_1(\theta)$ and $g(\alpha)$ is given by

$$\Pi_1(\theta, \alpha/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}\alpha^{(j+u)-1}(1-\alpha)^{(N-j+v)-1}\theta^{(t+a)-1}(1-\theta)^{mN-mj+b-1}}{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u,N-j+v)\, B(t+a,mN-mj+b)}$$

(1.33)

The marginal posterior distribution of θ and α are respectively given by

$$\Pi_1(\theta/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)\,\theta^{(t+a)-1}(1-\theta)^{mN-mj+b-1}}{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u,N-j+v)\, B(t+a,mN-mj+b)} \quad (1.34)$$

$$\Pi_1(\alpha/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j} B(t+a,mN-mj+b)\alpha^{(j+u)-1}(1-\alpha)^{(N-j+v)-1}}{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u,N-j+v)\, B(t+a,mN-mj+b)} \quad (1.35)$$

Similarly, the joint posterior p.d.f of θ and α corresponding to the prior $h_2(\theta)$ and $g(\alpha)$ is given by

$$\Pi_2(\theta, \alpha/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}\alpha^{(j+u)-1}(1-\alpha)^{(N-j+v)-1}\theta^{(t+a)-1}(1-\theta)^{mN-mj+b-1}e^{-c\theta}}{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u,N-j+v)\, B(t+a,mN-mj+b)M(t+a,a+b+mN-mj,-c)} \quad (1.36)$$

The marginal posterior distributions of θ and α are respectively given by

$$\Pi_2(\theta/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)\,\theta^{(t+a)-1}(1-\theta)^{mN-mj+b-1}e^{-c\theta}}{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u,N-j+v)\, B(j+a,mN-mj+b)M(t+a,a+b+mN-mj,-c)} \quad (1.37)$$

$$\Pi_2(\alpha/\underline{x}) = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j} B(t+a,mN-mj+b)M(t+a,a+b+mN-mj,-c)\alpha^{(j+u)-1}(1-\alpha)^{(N-j+v)-1}}{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u,N-j+v)\, B(t+a,mN-mj+b)M(t+a,a+b+mN-mj,-c)} \quad (1.38)$$

Under SELF, the Bayes estimate of θ^r and α^r corresponding to the posterior density (1.34) and (1.35) respectively, are given by

$$\hat{\theta}^r_{1B} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u,N-j+v)\, B(t+a+r\,,mN-mj+b)}{\sum_{j=0}^{N_0}\binom{N_0}{j}\, B(j+u,N-j+v)\, B(t+a\,,mN-mj+b)} \tag{1.39}$$

$$\hat{\alpha}^r_{1B} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u+r,N-j+v)\, B(t+a\,,mN-mj+b)}{\sum_{j=0}^{N_0}\binom{N_0}{j}\, B(j+u,N-j+v)\, B(t+a\,,mN-mj+b)} \tag{1.40}$$

Under WSELF, when $w(\theta) = \theta^{-2}, z(\alpha) = \alpha^{-2}$, the minimum expected loss (MEL) estimate of θ^r and α^r corresponding to the posterior density (1.34) and (1.35) respectively, are given by

$$\hat{\theta}^r_{1M} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u,N-j+v)\, B(t+a+r-2\,,mN-mj+b)}{\sum_{j=0}^{N_0}\binom{N_0}{j}\, B(j+u,N-j+v)\, B(t+a-2\,,mN-mj+b)} \tag{1.41}$$

$$\hat{\alpha}^r_{1M} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u-2+r,N-j+v)\, B(t+a\,,mN-mj+b)}{\sum_{j=0}^{N_0}\binom{N_0}{j}\, B(j+u-2,N-j+v)\, B(t+a\,,mN-mj+b)} \tag{1.42}$$

Finally under WSELF, when $w(\theta) = \theta^{-2}e^{-\delta\theta}; \delta > 0$, and $z(\alpha) = \alpha^{-2}e^{-\lambda\alpha}, \lambda > 0$, the EWMEL estimate of θ^r and α^r corresponding to the posterior density (1.34) and (1.35) respectively, are given by

$$\hat{\theta}^r_{1E} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u,N-j+v)\, B(t+a+r-2\,,mN-mj+b)M_1}{\sum_{j=0}^{N_0}\binom{N_0}{j}\, B(j+u,N-j+v)\, B(t+a-2\,,mN-mj+b)M_2} \tag{1.43}$$

where $M_1 = M(a + t - 2 + r, a + b + t + mN - mj - 2 + r, -\delta)$

$M_2 = M(a + t - 2, a + b + t + mN - mj - 2, -\delta)$

$$\hat{\alpha}^r_{1E} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u-2+r,N-j+v)\, B(t+a\,,mN-mj+b)M_3}{\sum_{j=0}^{N_0}\binom{N_0}{j}\, B(j+u-2,N-j+v)\, B(t+a\,,mN-mj+b)M_4} \tag{1.44}$$

where $M_3 = M(j + u - 2 + r, N + u + v - 2 + r, -\lambda)$

$M_4 = M(j + u - 2, N + u + v - 2, -\lambda)$

Also, SELF, the Bayes estimate of θ^r and of α^r corresponding to the posterior density (1.37) and (1.38) respectively, are given by

$$\hat{\theta}^r_{2B} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j} B(j+u,N-j+v)\, B(t+a+r\,,mN-mj+b)M_5}{\sum_{j=0}^{N_0}\binom{N_0}{j}\, B(j+u,N-j+v)\, B(t+a\,,mN-mj+b)M(t+a,a+b+mN-mj,-c)} \tag{1.45}$$

where, $M_5 = M(a + t + r, a + b + t + mN - mj + r, -c)$

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$$\hat{\alpha}^r_{2B} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u+r,N-j+v)\ B(t+a\ ,mN-mj+b)M(t+a,a+b+mN-mj,-c)}{\sum_{j=0}^{N_0}\binom{N_0}{j}\ B(j+u,N-j+v)\ B(t+a\ ,mN-mj+b)M(t+a,a+b+mN-mj,-c)} \qquad (1.46)$$

Under WSELF, when $w(\theta) = \theta^{-2}, z(\alpha) = \alpha^{-2}$, the MEL estimate of θ^r and α^r corresponding to the posterior density (1.37) and (1.38) respectively, are given by

$$\hat{\theta}^r_{2M} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)\ B(t+a+r-2\ ,mN-mj+b)M_6}{\sum_{j=0}^{N_0}\binom{N_0}{j}\ B(j+u,N-j+v)\ B(t+a-2\ ,mN-mj+b)M_7} \qquad (1.47)$$

where, $M_6 = M(a + t + r - 2, a + b + t + mN - mj + r - 2, -c)$,

$M_7 = M(a + t - 2, a + b + t + mN - mj - 2, -c)$

$$\hat{\alpha}^r_{2M} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u+r-2,N-j+v)\ B(t+a\ ,mN-mj+b)M(t+a,a+b+mN-mj,-c)}{\sum_{j=0}^{N_0}\binom{N_0}{j}\ B(j+u-2,N-j+v)\ B(t+a\ ,mN-mj+b)M(t+a,a+b+mN-mj,-c)} \qquad (1.48)$$

Finally under WSELF, when $w(\theta) = \theta^{-2}e^{-\delta\theta}; \delta > 0$, and $z(\alpha) = \alpha^{-2}e^{-\lambda\alpha}, \lambda > 0$, the EWMEL estimate θ^r and α^r corresponding to the posterior density (1.37) and (1.38) respectively, are given by

$$\hat{\theta}^r_{2E} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u,N-j+v)\ B(t+a+r-2\ ,mN-mj+b)M_8}{\sum_{j=0}^{N_0}\binom{N_0}{j}\ B(j+u,N-j+v)\ B(t+a-2\ ,mN-mj+b)M_9} \qquad (1.49)$$

where $M_8 = M(a + t - 2 + r, a + b + t + mN - mj - 2 + r, -(c + \delta))$

$M_9 = M(a + t - 2, a + b + t + mN - mj - 2, -(c + \delta))$

$$\hat{\alpha}^r_{2M} = \frac{\sum_{j=0}^{N_0}\binom{N_0}{j}B(j+u+r-2,N-j+v)\ B(t+a\ ,mN-mj+b)M(t+a,a+b+mN-mj,-c)M_{10}}{\sum_{j=0}^{N_0}\binom{N_0}{j}\ B(j+u-2,N-j+v)\ B(t+a\ ,mN-mj+b)M(t+a,a+b+mN-mj,-c)M_{11}}$$

(1.50)

$M_{10} = M(j + u + r - 2, N + u + v + r - 2, -\lambda)$

$M_{11} = M(j + u - 2, N + u + v - 2, -\lambda)$

An Illustrative Example

In order to demonstrate the practical applications of the above-mentioned results, we fitted the classical Poisson distribution and zero-inflated Poisson distribution to the data pertaining to the number of strikes in 4-weaks in Vehicle Manufacturing Industry in the United Kingdom during 1948-1958 (Kendall (1961)). The expected frequencies of classical Poisson distribution are obtained by maximum likelihood estimator, while the expected frequencies of zero-inflated

Poisson distribution are obtained by using Bayes estimators, obtained under square error loss function (SELF) and two different weighted square error loss functions. The prior values used for the beta distribution (2.2) will be $u = 3, v = 1$, while those used for the gamma distribution (3.3) will be $a = 0.25$, $b = 1$ and for exponentially weighted minimum expected loss (EWMEL) estimates (3.11) and (3.12) will be $\delta = \lambda = 0.25$. The values for the prior parameters $a, b, u, v, \delta, \lambda$ were chosen so that the posterior distribution would reflect the data as much, and the prior information as little, as possible. The observed frequencies, expected frequencies, the value of Pearson's chi-square statistics is given in table-I.

Table 1: *Number of outbreaks of Strike in Vehicle manufacturing Industry in the U.K. during 1948-1958*

No. of Outbreaks	Observed Frequency	Expected Frequency (Poisson Distribution)	Expected Frequency (Zero-inflated Poisson Distribution)		
			SELF	WSELF	
				MEL	EWMEL
0	110	103.5	110.3	107.9	108.0
1	33	42.5	30.4	33.1	33.1
2	9	8.7	11.7	11.7	11.7
3	3	1.2	3.0	2.8	2.7
4	1	0.1	0.6	0.5	0.5
Total	156	156	156	156	156
χ^2		3.4317	0.5690	0.3079	0.2796
Estimated value					
θ		0.4103	0.7673	0.7089	0.7038
α			0.4532	0.3927	0.3910

Conclusion and Comments

The values of the expected frequencies and the corresponding χ^2 value clearly shows that the zero-inflated Poisson distribution provided a closer fit than that provided by the classical Poisson distribution. It is also clear from the table that the Bayes estimators obtained under weighted squared error loss functions (WSELF) gives closer fits than the Bayes estimator obtained under squared error loss function (SELF). Also, the exponentially minimum expected loss (EWMEL) estimates gives better fits than the minimum expected loss (MEL) estimates. Keeping in view the importance of count data modeling it is recommended that whenever the experimental number of zeros are more than that given by the model, the model should be adjusted accordingly to account for the extra zeros.

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