MOMENT- CURVATURE RELATION FOR BAMBOO REINFORCED CONCRETE SECTIONS

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ABSTRACT

The demand for building materials has surged globally where the global economy revolves around the construction industry, significantly leading to resource depletion and environmental exploitation. Concrete is the second most consumed product globally, after clean water, with 3 tonnes used annually per person (C.R. Gagg [1]). With soaring steel prices and its large carbon footprint, there is a pressing need for sustainable alternatives. Bamboo, being far less expensive than steel and emitting significantly less carbon dioxide during production, presents an economically viable and environmentally friendly option for construction. The utilization of bamboo as reinforcement in reinforced concrete (RC) is in its nascent stages since the bamboo reinforced concrete (BRC) members involves more uncertainties compared to steel-reinforced concrete (SRC) members. Therefore, exploring the feasibility of bamboo as a reinforcement material is essential for the emergence of a more sustainable construction industry. Ductility is crucial for seismic resistant design of RC structures. When the structure is loaded beyond the yielding stage, plastic hinges are formed at specific locations of the structural members. The non-linear behaviour is expressed in terms of the moment-curvature (M-C) relationship, which is determined from the cross-sectional behaviour of these members. The M-C relationship can be evaluated both experimentally and numerically. Experimental studies are the best way to determine the cross-sectional properties. However, it is not practically feasible to develop 1:1 scale model for RC structures. Therefore, numerical iteration methods and finite element programs can be used to determine the M-C relationship. This study focused on developing the M-C relationship of BRC beams and columns numerically. This work involved developing a code for the M-C relation for BRC Beam and BRC column based on the σ-ε relations of bamboo splint and ordinary concrete. Ductility factors were determined from the M-C plots. Although BRC structural elements exhibited lower ductility than conventional SRC members, they still demonstrate significant ductile capacity.

1. Introduction

Bamboo has emerged as a potential reinforcement material in concrete structures because of its high tensile strength to weight ratio, low carbon footprint and being a green material. In addition to this being a grass, bamboo is locally available in tropical region. Also, it grows rapidly, making it more economically viable. India has annual bamboo production as 4.5 million tonnes.). The utilization of bamboo as a substitute for steel reinforcement in concrete is in the initial stages of exploration as bamboo reinforced concrete (BRC) members involve a lot more uncertainties than steel reinforced concrete (SRC) members. Steel is priced at Rs 72 /kg, while bamboo costs only Rs 2.5/kg. Building a house with steel amounts to Rs 1800 /ft^2. In contrast, bamboo reduces the cost to approximately Rs 600-700 /ft^2, representing a substantial one-third cost reduction. In addition to this escalating price of steel, the manufacturing of 1 kg of steel produces 1.85 tons of CO_2, which is almost 8% of global carbon dioxide emissions, while bamboo emits just 80

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times less carbon dioxide [2]. In Maharashtra bamboo production is 2,47,239 tones. The Konkan region contributes 70,000 tonnes of bamboo production [3]. The widespread structural applications of bamboo particularly in seismic regions is hindered by lack of comprehensive understanding of its structural behaviour. Hence a detailed study on its moment curvature relationship and ductility characteristics is needed.

Moment-curvature (M-C) can be determined through experimental or numerical methods. While experimental methods provide the most accurate representation of the M-C relationship, they are often impractical due to cost, time, and scale limitations, leading to the use of numerical methods and finite element software to define the M-C relationship. Analytically the M-C relationship is determined through an iterative process assuming a strain distribution across the section. The stresses in concrete and reinforcement are calculated using their respective constitutive models. The internal forces are computed subsequently equilibrium is checked. Once the equilibrium is achieved, the moment and corresponding curvature are computed. Otherwise, the strain distribution is revised and the iterative process is repeated. The M-C relationship also give quantitative measure of strength reduction beyond the peak point and degradation of the flexural rigidity

Studies in the field of BRC was initiated by Ghavami [4] researching on lightweight concrete beams reinforced with bamboo. This research demonstrated a significant 400% increase in the ultimate applied load of BRC beams compared to unreinforced concrete. Building upon this foundation, Ghavami [5] expanded the research scope, investigating various impermeability treatments to address bamboo's water absorption issues and further improvement in its bonding with concrete with Sikadur 32-Gel. Agarwal et al.,[6] conducted experimental investigations on chemically treated BRC beams and columns. They observed that BRC elements exhibited satisfactory load-carrying capacity but lower ductility compared to SRC. The flexural behavior of concrete beams reinforced with bamboo strips was investigated by Mali & Datta [7]. Their studies showed that beams reinforced with bamboo had higher shear and flexural capacity than PCC beams, and lower shear and flexural capacity that beams with conventional reinforcement. Several researches were conducted to establish flexural, shear and bond properties of BRC. Terai & Minami [8] studied the feasibility of using bamboo as reinforcement in concrete. They conducted beam tests to assess the flexural cracking and shear cracking strength of bamboo. Additionally, monotonic compression tests were carried out to investigate the fracture behavior and mechanical properties of bamboo when used to reinforce concrete. The results revealed that bamboo-confined concrete specimens exhibited a more ductile and complex behavior compared to plain, unconfined columns.

Pramodini and Satish[9] recommended a detailed procedure using VBA in Microsoft Excel for computing moment curvature values of RC columns based on IS456:2000 [10]. The M-C relationships for the SRC members are plotted for various cross section of columns members. These pre-defined plastic hinges can be given as input for the push over analysis when studying the structure subjected to non-linear time varying loads. This study involves developing a Python code for the M-C relation for BRC Beam and BRC column. The algorithm was developed based on the σ-ε relation of bamboo splint, for the point plasticity idealisation of sections in the non-linear static analysis of BRC Structures. This work also includes the analysis of ductility characteristics of BRC elements compared to conventional SRC.

2. Stress-strain curve for concrete, steel and bamboo

The design stress strain curve of concrete as specified in IS 456:2000 is considered for the determination of compressive stress [10]. The design stress-strain curve of concrete in flexural compression is shown in Figure 1. The stress strain plot is idealized as parabolic in the initial portion up to a yield strain of 0.002

and a straight line of constant stress 0.67 f_{ck} until ultimate strain of 0.0035. The value of maximum compressive stress of concrete in the curve is restricted to 0.67 f_{ck} considering the difference in the compressive strength of concrete in actual concrete structure and that obtained from a uniaxial compression test for the same quality of concrete. The code limits the design stress of concrete as 0.45 f_{ck} considering a partial safety factor $\gamma_c = 1.5$ in Limit state method of design (LSD).

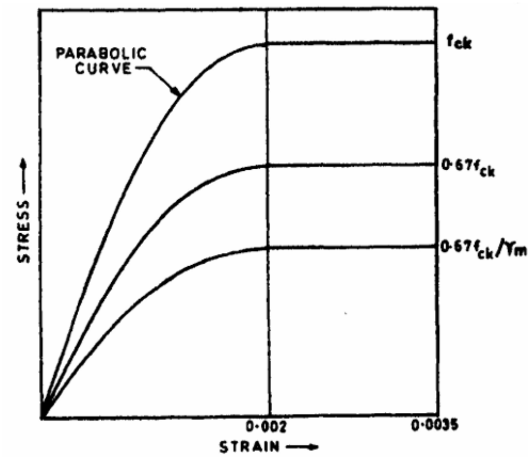

Figure 1. Design stress-strain curve of concrete (IS 456:2000, Figure 21[10])

In the case of conventional steel reinforcement, the increase in strength beyond the yield point (due to strain-hardening) is generally ignored for design purposes. The design stress strain curve is idealised as an ideal elasto-plastic stress-strain curve (with an initial linearly elastic line up to yield, followed by a line at constant stress, denoting the post-yielding behaviour). A partial safety factor $\gamma_m = 1.15$ is applied to characteristic strength of steel to obtain design strength in Limit state method of design (LSD). A typical design stress strain curve of steel reinforcement is shown in Figure 2. In the absence of a definite yield point as in the case of HYSD bars, the 0.2 percent proof stress is generally taken as the yield strength.

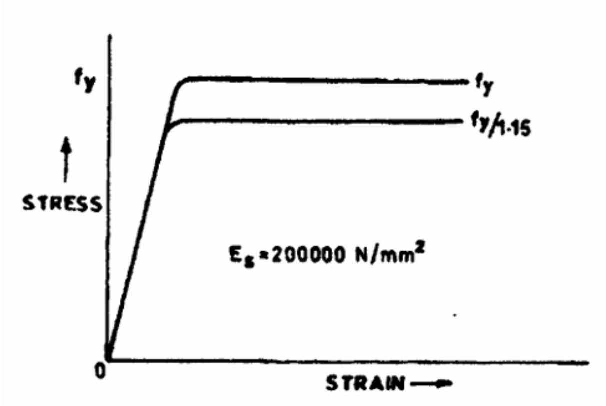

Figure 2. Design stress strain curve of steel reinforcement. (IS 456:2000, Figure 23B[10])

Unlike steel a standardized design stress strain plot is not available for bamboo, since its mechanical properties varies from species to species, culm position, age and even from location to location. The tensile testing of locally available six species of bamboo is carried out in accordance with IS 6874:2008[11]. Three samples of each species are prepared and tested in displacement controlled UTM. The stress strain curves of all six species obtained experimentally are shown in Figure 3. The maximum tensile strength exhibited by bamboo ranges between 100 Mpa to 250 Mpa, which is comparatively less than steel reinforcement. Hence more area of reinforcement is required to achieve the same moment capacity.

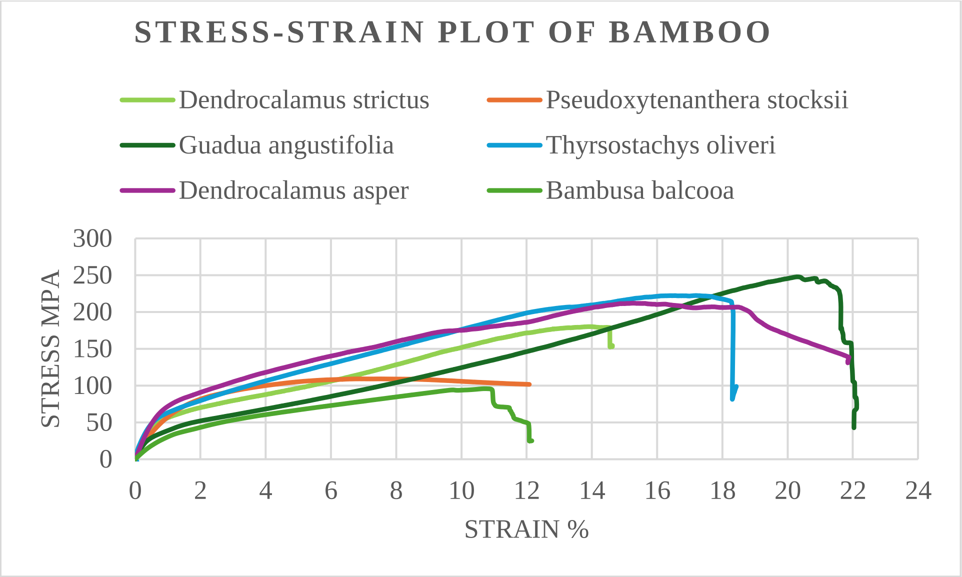

Figure 3. Stress strain curve of Bamboos

Among the six species, Bambusa balcooa demonstrated lower tensile strength, with maximum stress values ranging from 90 to 100 MPa. Figure 4 depicts the experimental stress strain plot of three samples of B. balcooa. One notable characteristic of B. balcooa is its consistency across specimens, with all three samples showing almost similar stress-strain curves. In terms of flexibility, B. balcooa shows respectable performance with strains at failure between 12% and 20%. Considering this lower variability, it can be assumed that B. balcooa offer reliable and predictable mechanical properties, which can be advantageous in engineering applications.

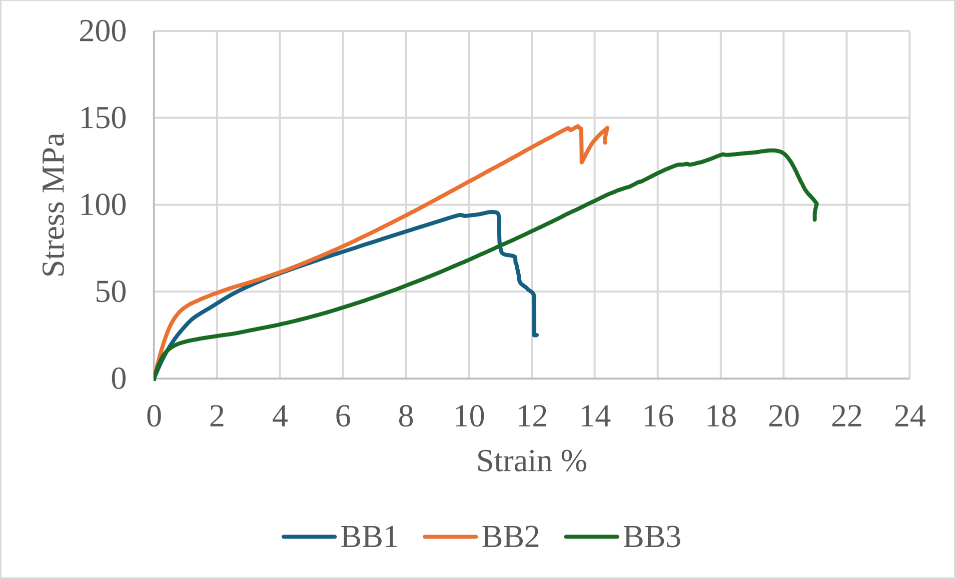

Figure 4. Stress strain plot of Bambusa balcooa

Among these three-test specimen, the stress-strain plot of BB1 is considered for developing M-C relationship since it represents the lowest strength and ductility among the three.

3. Moment-curvature relationship

The moment-curvature (M-C) relationship is essential for performing non-linear analysis of structures, particularly when subjected to seismic loads and it provides member ductility which helps in obtaining the structure ductility from the analysis. This relationship forms the basis for the non-linear behavior of RC members and is essential for predicting the performance of structures under various loading conditions. For buildings located in seismic prone areas, the ability of structures to dissipate energy through inelastic deformations is paramount. This ductile behavior is achieved by the formation of plastic hinges at specific locations within the structure. The available ductility in these RC hinges is directly related to the M-C relationship of the structural elements. The M-C relationship is a graphical representation of the bending moment (M) versus the curvature (C) of a cross-section. Curvature (C) is a geometrical parameter representing cross sectional deformation and is defined as the rate of change of the slope of the neutral axis along the length of the member. The M-C relationship also give a clear view of strength reduction beyond the peak point and degradation of the flexural rigidity.

Ductility, defined as the capacity of a material to undergo plastic deformation without fracture, is directly reflected in the post-yield portion of the MC curve. A longer, more gradual post-yield curve indicates a higher energy dissipation capacity in a more ductile element. The ductility factor, often calculated as the ratio of ultimate to yield curvatures, can be derived from key points on the MC curve, thus quantifying the element's ability to sustain large deformations without significant loss of strength. Therefore, it is considered as a crucial property for seismic design.

4. Methodology

The assumptions of flexural design of RC sections based on IS456:2000 is taken into consideration [10]. The maximum compressive strain in extreme concrete compression fiber is taken as 0.0035. The ultimate tensile strength of the B. balcooa is taken as 95.895 N/mm^2. As per IS 15912: 2018[12], the neutral axis depth for balanced section is considered as

$$x_u = \begin{cases} \text{0.25d to 0.254d for M15} \\ \text{0.274d to 0.278d for M20} \end{cases}$$

The algorithm for the development of code is developed based on Park & Paulay [13]. The proposed algorithm is depicted in Figure 5. This process involves incrementing the value of strain in extreme compression fiber of concrete upto 0.0035. Assume a suitable value of neutral axis depth. The value of strains at the level of each bamboo bars were calculated using the equation:

$$\varepsilon_{bi} = \varepsilon_c \frac{x_u - d_i}{x_u}$$

The corresponding stress in bamboo bars are interpolated from the stress strain data of the bamboo. The total tensile as well as compressive forces are calculated and the force equilibrium are checked for each value of neutral axis. If equilibrium is not obtained the neutral axis value is modified. Moment and curvature are calculated for the particular values of assumed strain (ranging from 0.001 to 0.0035) and the neutral axis depth in which force equilibrium is achieved. The iterative procedure is continued by incrementing the value of strain. From the plot of M-C, the yield moment, yield curvature, ultimate moment and ultimate curvature are noted. From these observations, ductility is calculated. Yield moment is the moment at which tension reinforcement reaches its yield strength. The curvature corresponding to this moment is taken as yield curvature. Ductility factor is the quantitative measure of a beam's ability to undergo plastic deformation without failure. It is calculated as the ratio of ultimate curvature to yield curvature.

$$\text{Curvature } \phi = {}^{\varepsilon_c}/_{x_u}$$

$$\text{Ductility factor} = \phi_u \;/\; \phi_y$$

where, ϕ_u −ultimate curvature and ϕ_y- yield curvature

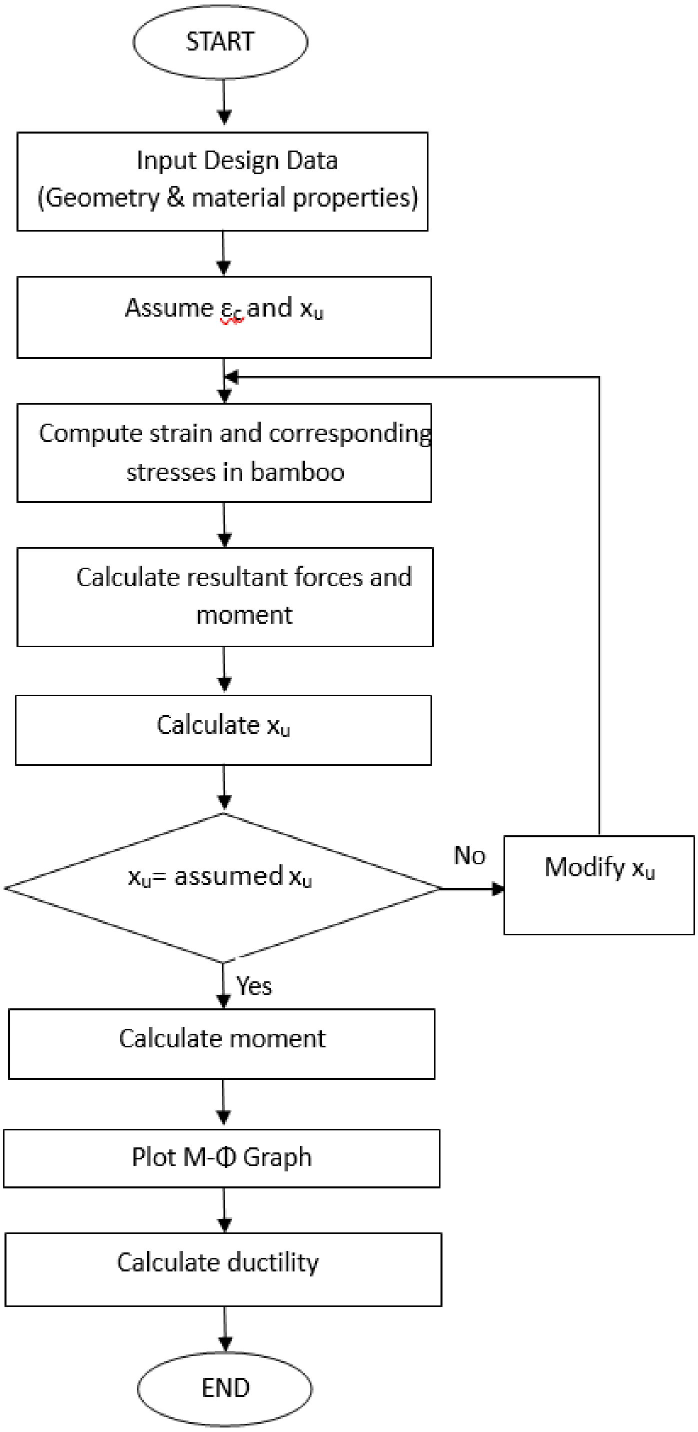

Figure 5. Algorithm to derive M-C relationship of BRC structural elements.

5. Validation of the Algorithm

The proposed algorithm is validated against the example 4.16 given in Pillai and Menon [14]. A doubly reinforced beam measuring 300mm x 700 mm, reinforced with 4 number of 25 mm diameter bars at tension zone and 2 numbers of 25mm diameter bars at compression zone is used as a test case. M20 grade of concrete and Fe 415 grade of steel are considered. The salient points of stress strain data of Fe 415 steel used for analysis are taken from Table 3.2 of Pillai and Menon [14]. It can be inferred from the Figure 6 that the results generated using the proposed algorithm are in good agreement with exact solution obtained by considering strain compatibility in solved example given in Pillai and Menon[14]. This high degree of concordance in both ultimate curvature and moment calculations validates the accuracy and reliability of the proposed algorithm for analyzing BRC beams

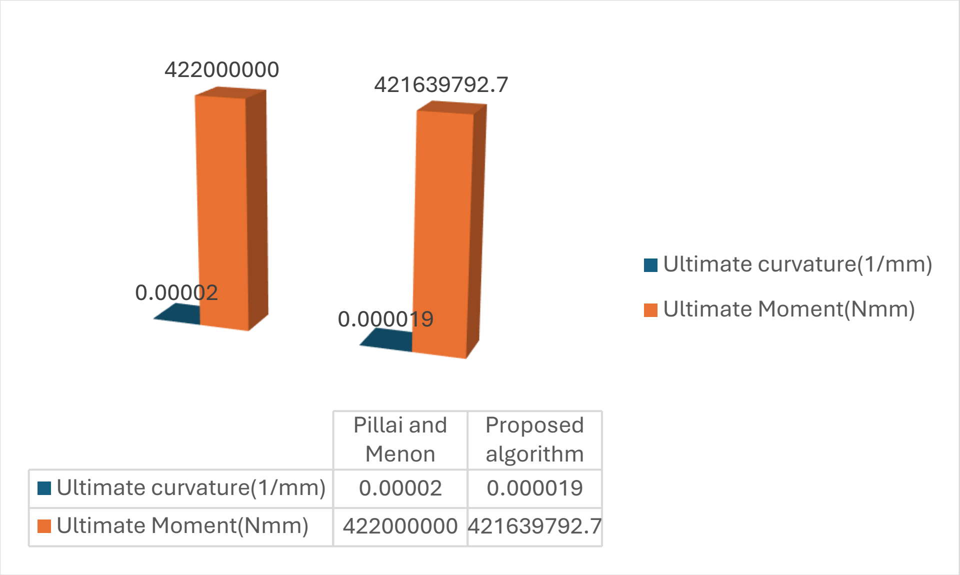

Figure 6. Comparison of Ultimate Moment and Curvature: Pillai and Menon[14] v/s. Proposed Algorithm

6. Input design data for M-C relationship

Three structural elements are selected for these studies: singly and doubly reinforced BRC beams and BRC columns. M20 grade concrete and Bambusa balcooa of tensile strength 95.895 N/mm^2 are used for the analysis. Bamboo splints of cross-sectional dimension 15mm x 15mm is provided as reinforcement. The axial load acting on BRC column is assumed to be 60kN. The numerical code is developed using Python software. The various inputs to the code are summarized in Table 1.

Table 1. Input design data for M-C relationship

Parameters	Singly Reinforced BRC beam	Doubly Reinforced BRC beam	BRC column
Width of cross-section (mm)	150	200	200
Overall depth of cross-section(mm)	250	300	200
Clear cover (mm)	30	30	40
Size of reinforcement (mm)	15 x 15	15 x 15*	15 x 15

Number of reinforcements	5	2 at top 4 at bottom	8 splints in 3 rows
Characteristics strength of concrete (N/mm²)	20	20	20
Tensile strength of bamboo (N/mm²)	95.895	95.895	95.895

***** (both in compression and tension)

7. Results and Discussions

The M-C plots are obtained for three cases: a) Singly reinforced BRC beam, b) doubly reinforced BRC beam and c) BRC Column. Figure 7 depicts the M-C plot of singly reinforced BRC beams. It exhibits a nonlinear behaviour similar to that of typical SRC beam. The significant curvature increase in the post-yield phase indicates good ductility.

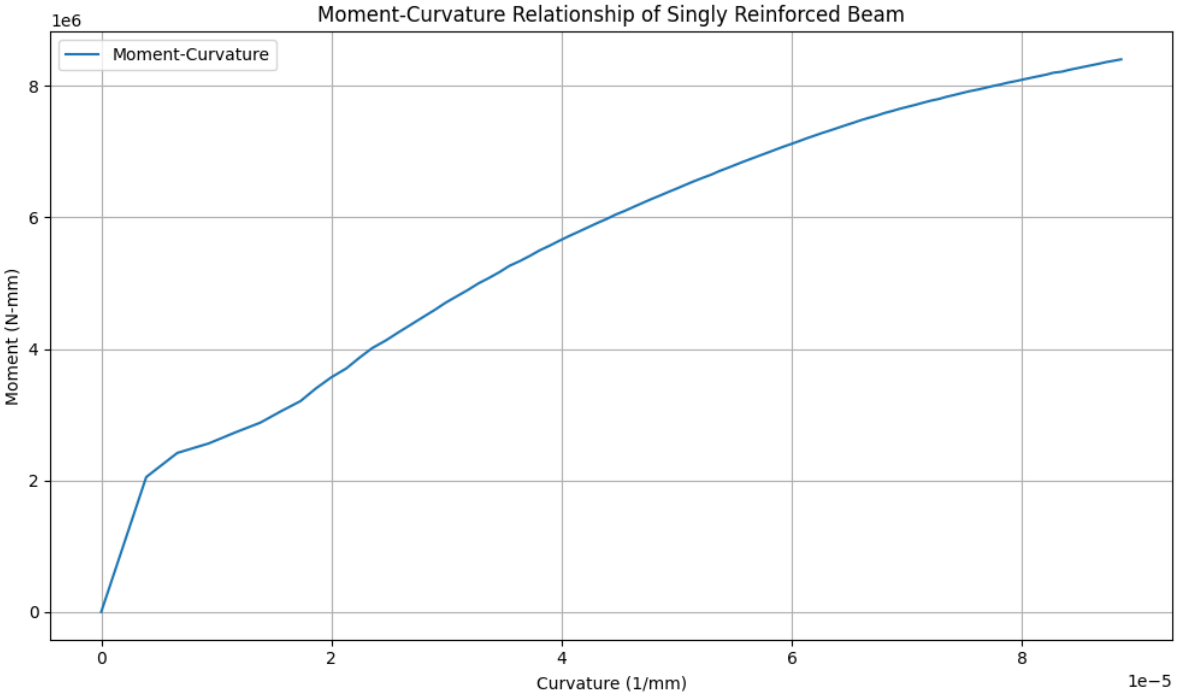

Figure 7. Moment curvature relationship for Singly reinforced beam

The MC plot of doubly reinforced beam can be idealized as a bi-linear plot as shown in Figure 8. The initial linear elastic phase was followed by a post-yield phase with significant curvature increase as shown in plot. The MC relationship for the doubly reinforced beam shows a relatively linear initial response up to a moment of about 0.6 kN-m and a curvature of approximately 3x10-5 1/mm. The compression reinforcement attributes to increased moment capacity, enhanced post-yield stiffness but slightly lower ultimate curvature.

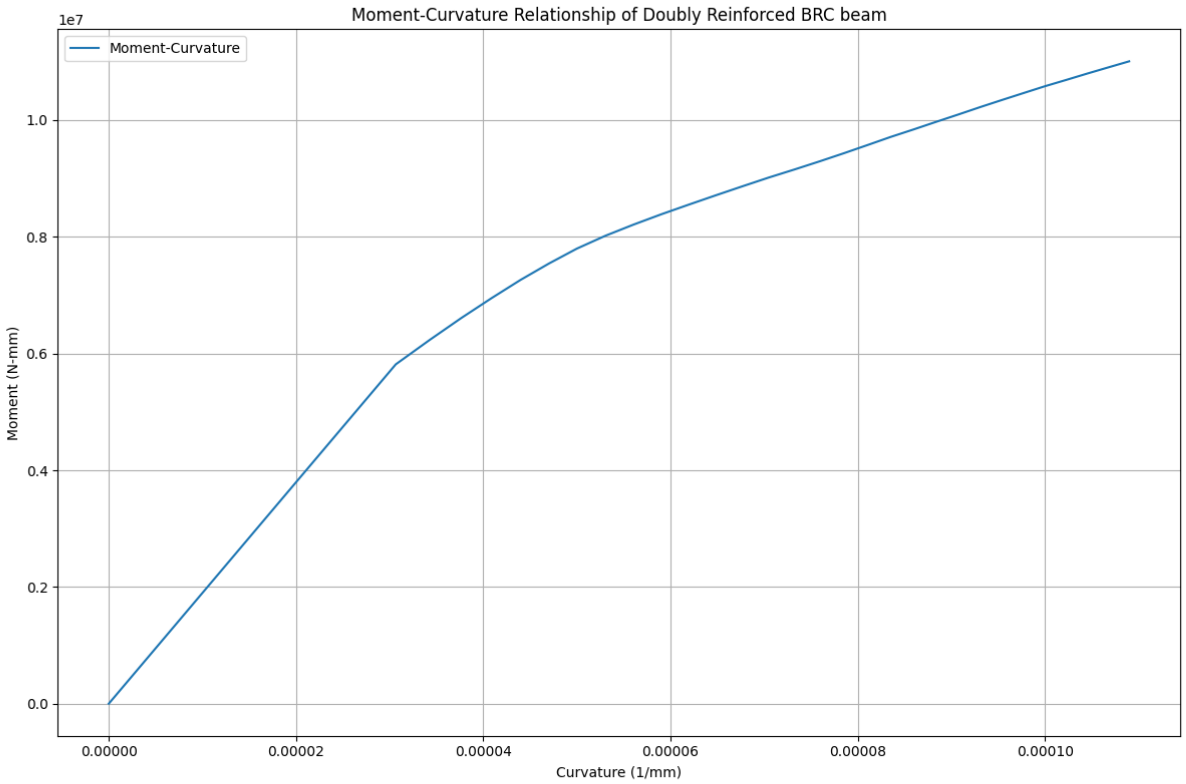

Figure 8. Moment curvature relationship for Doubly reinforced beam

The M-C plot of BRC Column exhibited a more pronounced plateau in the post yield region as shown in Figure 9. The plot suggests that BRC columns can undergo significant deformation without sudden failure. The high ductility of BRC columns make them particularly suitable for structures in seismically active regions.

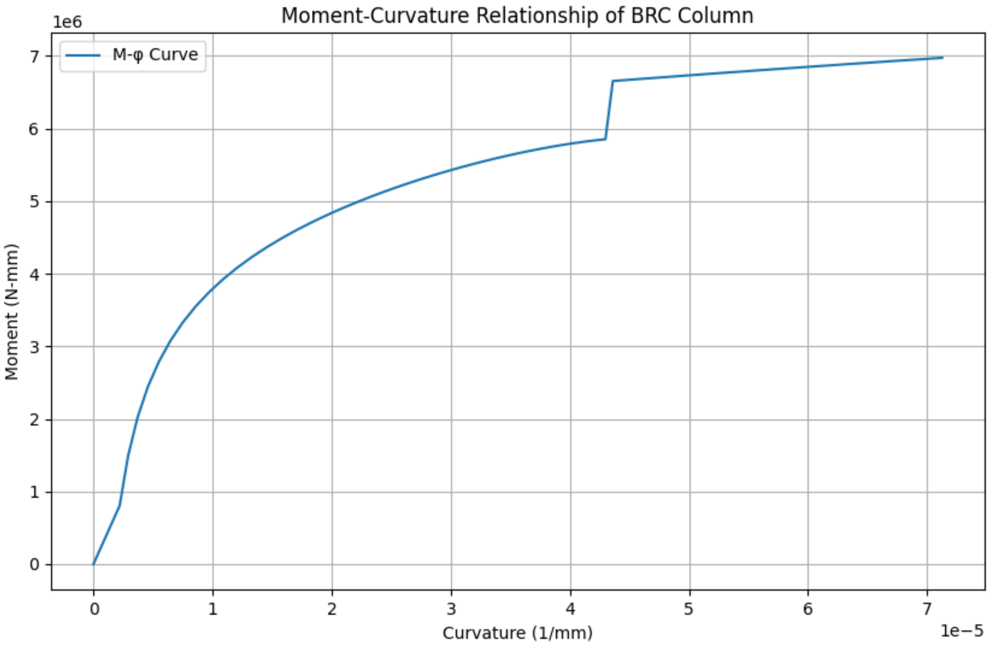

Figure 9. Moment curvature relationship for BRC Column

The moment and curvature corresponding to yield point are determined as yield moment and yield curvature. The ultimate moment and ultimate curvatures of the three structural elements are also tabulated. Ductility is determined as the ratio of ultimate curvature to yield curvature. They provide a quantitative measure of ability to undergo inelastic deformation. The ductility factors obtained in all the cases of structural elements are summarized in Table 2.

Table 2. BRC Structural Element Performance: Moments, Curvatures, and Ductility

Structural element	Yield Moment (kNm)	Yield curvature (1/mm)	Ultimate moment (kNm)	Ultimate curvature	Ductility factors
Singly reinforced beam	6.279	0.000048	8.405	0.000089	1.854
Doubly reinforced beam	8.22	0.000056	11.007	0.000109	1.946
column	6.675	0.000045	6.972	0.000071	1.577

The overall shapes of the moment-curvature relationships developed exhibits unique mechanical behaviors distinct from traditional steel-reinforced concrete. Ghavami (2005) previously highlighted bamboo's potential in structural applications, which is substantiated by these experimental results. The lower moment capacities of BRC structural elements compared to SRC can be attributed to the lower tensile strength and elastic modulus of bamboo compared to steel. Nevertheless, the obtained plots provide empirical evidence supporting bamboo's structural potential, particularly for low-rise earthquake-resistant designs, while underscoring the necessity for specialized design guidelines that comprehensively address bamboo's unique mechanical characteristics.

8. Conclusions

The comprehensive investigation into Bamboo-Reinforced Concrete (BRC) structural elements unveils critical insights into their seismic performance and structural characteristics. The derived moment-curvature plots and ductility factors reveal that BRC elements, particularly columns, demonstrate significant potential for low-rise earthquake-resistant structures, with ductility factors approaching those of Steel-Reinforced Concrete (SRC) systems. While the moment capacities are lower due to bamboo's inherently reduced tensile strength and elastic modulus, requiring approximately 3.5 times more reinforcement area compared to steel to achieve equivalent structural performance, the material offers compelling advantages in terms of cost-effectiveness and environmental sustainability. The study's detailed moment-curvature relationships provide a foundational framework for performance-based seismic design approaches specifically tailored to BRC structures, enabling more accurate predictions of structural behavior under varying earthquake intensities. Despite the promising findings, the research underscores the need for future investigations to develop comprehensive seismic design guidelines that can fully leverage the unique properties of bamboo as a construction reinforcement material, bridging the current knowledge gap and paving the way for more innovative and sustainable structural design practices.

Conflict of interest

The author(s) declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this paper.

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