Inverse Majority Neighborhood Number for Tensor Product of Path and Cycle

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ABSTRACT

The tensor product of G and H is the graph, denoted as $G \otimes H$, whose vertex set is $V(G) \otimes V(H)$ and for which vertices (g, h) and (g', h') are adjacent if $gg' \in E(G)$ and $hh' \in E(H)$[1]. In this article the inverse majority neighborhood number $n_i^{-1}(\chi)$ of G for the tensor product of path and cycle are determined. Also obtained some bounds for $\chi = P_n \otimes C_l$.

Keywords: Neighborhood set, Majority Neighborhood set, Majority Neighborhood number, Inverse Majority neighborhood number.

1 Introduction

In this paper $G = (V, E)$ be a simple, undirected and connected graph with p vertices and q edges.A set S of vertices in a graph $G = (V, E)$ is a neighborhood set of G if $G = \cup_{v\in S}\langle N[v]\rangle$. The neighbourhood number $n(G)$ of G is the minimum cardinality of a neighborhood set of G. A neighborhood set S of a graph G such that $|S| = n(G)$ is called a minimum neighborhood set of G [1-4].Let S be a minimum neighborhood set of G. If $V - S$ contains a neighborhood set $S'$of G, then S' is called an inverse neighbourhood set of G with respect to S. The inverse neighborhood number $n^{-1}(G)$ of G is the minimum cardinality of an inverse neighbourhood set of G[15].The idea of majority neighborhood set has been studied by Prof. Swaminathan V and J. Joseline Manora [6-10].The Neighborhood parameters studied very well inside the articles [5, 11-14,16-19] .

2 Inverse Neighborhood number for the tensor product of Path and Cycle

Definition 2.1

If S_M be the majority neighborhood set of G. If $V - S_M$ contains a majority neighbourhood set S'_M of G, then S'_M is called an inverse majority neighborhood set of G with respect to S_M. The inverse majority neighbourhood number $n_M^{-1}(G)$ of G is the minimum cardinality of an inverse majority neighborhood set of G [16].

Theorem 2.1 Let $\chi = P_2 \otimes C_l$ be the graph with $l \geq 3$, then $n_i^{-1}(\chi) = \left\lceil \frac{l}{2} \right\rceil$.

Proof. Let $\chi = P_2 \otimes C_l$ with $l \geq 3$ and $\psi(\chi) = \{e_{1,1}, e_{1,2}, e_{1,3}, \dots, e_{1,l}, e_{2,1}, e_{2,2}, \dots, e_{2,l}\}$ be the vertex set with $|\psi(\chi)| = |\varphi(\chi)| = 2l$. The graph $P_2 \otimes C_l$ have 2-regular graph. Let $S_M \in \psi(\chi)$ be the majority neighborhood set with $|S_M| = \left\lceil \frac{l}{2} \right\rceil$ and $S'_M \in \{\psi - S_M\} = \{e_{2,1}, e_{2,2}, \dots, v_{2,\left\lceil \frac{l}{2} \right\rceil}\}$. S'_M covers the edges $|\langle N[S'_M]\rangle| = 2\left\lceil \frac{l}{2} \right\rceil \geq \left\lceil \frac{q}{2} \right\rceil$. Therefore, S'_M is an inverse majority neighborhood set. Suppose $|S'_M| - 1 = \left\lceil \frac{l}{2} \right\rceil - 1 < \left\lceil \frac{q}{2} \right\rceil$. This set is not an inverse majority neighborhood set. Hence, $|S'_M| = n_i^{-1}(\chi) = \left\lceil \frac{l}{2} \right\rceil$.

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Theorem 2.2 Let $\chi = P_3 \otimes C_l$ be the graph with $l \geq 3$, then $n_i^{-1}(\chi) = \left\lceil \frac{l}{2} \right\rceil$.

Proof. Let $\chi = P_3 \otimes C_l$ with $n = 3, l \geq 3$ and $\psi(\chi) = \{e_{1,1}e_{1,2}, \dots, e_{1,l}, e_{2,1}, e_{2,2} \dots, e_{2,l}, e_{3,1}, e_{3,2}, \dots, e_{3,l}\}$ be the vertex set with $|\psi(\chi)| = 3l$ and $|\varphi(\chi)| = 4l$.

Case (i): $l = 3$.

Let $S_M = \{e_{2,3}, e_{1,3}\} \in \psi(\chi)$ be the majority neighborhood set with $|S_M| = \left\lceil \frac{l}{2} \right\rceil$

and$S_M' = \{e_{2,1}, e_{2,2}\}$. The set S_M'covers exactly $2l + 2$ edges and $2l + 2$ vertices. That is, $|\langle N[S_M'] \rangle| = 2l + 2 > \left\lceil \frac{q}{2} \right\rceil$ and $|N[S_M']|$=$2l + 2 > \left\lceil \frac{p}{2} \right\rceil$.

Therefore, S_M' is the inverse majority neighborhood set. Hence, $|S_M'| = \left\lceil \frac{l}{2} \right\rceil$. Suppose $|S_M'| - 1 = \left\lceil \frac{l}{2} \right\rceil - 1$ and $|\langle N[S_M'] \rangle| = (2l + 2) - 4 < \left\lceil \frac{q}{2} \right\rceil$. Therefore, S_M' is not an inverse majority neighborhood set. Hence $n_i^{-1}(\chi) = |S_M'| = \left\lceil \frac{l}{2} \right\rceil$.

Therefore, S_M' is the inverse majority neighborhood set. Hence, $|S_M'| = \left\lceil \frac{m}{2} \right\rceil$. Suppose $|S_M'| - 1 = \left\lceil \frac{m}{2} \right\rceil - 1$ and $|\langle N[S_M'] \rangle| = (2m + 2) - 4 < \left\lceil \frac{q}{2} \right\rceil$. Therefore, S_M' is not an inverse majority neighborhood set.

Case (ii): $l > 3$.

Choose $S_M = \{e_{2,1}, e_{2,2}, \dots, e_{2,\left\lceil \frac{l}{2} \right\rceil - 1}, e_{1,2}\}$ when l is odd and

$S_M = \{e_{2,1}e_{2,2}, \dots, e_{2,\left\lceil \frac{l}{2} \right\rceil}\}$ when l is even. In both the above cases,

$|S_M| = \left\lceil \frac{l}{2} \right\rceil$. Now choose $S_M' = \{e_{2,\left\lceil \frac{l}{2} \right\rceil + 1}, e_{2,\left\lceil \frac{l}{2} \right\rceil + 2}, \dots, e_{2,l}\}$ with respect to S_M, when l is even or odd. Each vertex S_M' exactly covers $2n + 2$ edges.

That is, $|\langle N[S_M'] \rangle| = (2n + 2) \left\lceil \frac{l}{2} \right\rceil > \left\lceil \frac{q}{2} \right\rceil = \left\lceil \frac{4l}{2} \right\rceil = 2l$ and the vertices covers$|N[S_M']| = (n + 2)\left(\left\lceil \frac{l}{2} \right\rceil - \left(\left\lceil \frac{l}{2} \right\rceil - 2\right)\right) + n\left(\left\lceil \frac{l}{2} \right\rceil - 2\right) > \left\lceil \frac{p}{2} \right\rceil = \left\lceil \frac{3l}{2} \right\rceil$. Therefore, S_M' is an inverse majority neighborhood set.

Suppose $|S_M'| - 1 = \left\lceil \frac{l}{2} \right\rceil - 1$ and $|\langle N[S_M'] \rangle| = (2n + 2) \left\lceil \frac{l}{2} \right\rceil - (n + 1) < \left\lceil \frac{q}{2} \right\rceil = \left\lceil \frac{4l}{2} \right\rceil = 2l$.Therefore S_M' is not an inverse majority neighborhood set. Hence, $n_i^{-1}(\chi) = |S_M'| = \left\lceil \frac{l}{2} \right\rceil$.

Theorem 2.3 Let $\chi = P_4 \otimes C_l$ be the graph with $l \geq 3$, then $n_i^{-1}(\chi) = \left\lceil \frac{3l}{4} \right\rceil$.

Proof. Let $\chi = P_4 \otimes C_l$ with $l \geq 3$ and $\psi(\chi) = \{e_{1,1}e_{1,2}, \dots, e_{1,l}, e_{2,1}, e_{2,2} \dots, e_{2,l}, e_{3,1}, e_{3,2}, \dots, e_{3,l}, e_{4,1}, e_{4,2}, \dots, e_{4,l}\}$ be the vertex set with $|\psi(\chi)| = 4l$ and $|\varphi(\chi)| = 6l$.

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$$deg(e_{1,1}) = \deg(e_{1,l}) = \deg(e_{4,1})\deg(e_{4,l}) = 2.$$

$deg(e_{2,1}) = deg\ (e_{2,l}) = deg(e_{3,1}) = deg(e_{3,l}) = 4$. Let $S_M = \left\{e_{2,1}, e_{2,2}, \dots, e_{2,\left(l-\left\lfloor\frac{l}{4}\right\rfloor\right)}\right\}$ be the majority neighborhood set with $|S_M| = \left\lceil\frac{3l}{4}\right\rceil$ and $S'_M \in \{\psi - S_M\} = \{e_{3,1}, e_{3,2}, \dots, e_{3,\left(l-\left\lfloor\frac{l}{4}\right\rfloor\right)}\}$. S'_M covers the edges $|\langle N[S'_M]\rangle| = 4\left(l - \left\lfloor\frac{l}{4}\right\rfloor\right) \geq \left\lceil\frac{q}{2}\right\rceil = \left\lceil\frac{6l}{2}\right\rceil = 3l$. Therefore, S'_M is an inverse majority neighborhood set. Suppose $|S'_M| - 1 = \left(l - \left\lfloor\frac{l}{4}\right\rfloor\right) - 1 = \left\lceil\frac{3l}{4}\right\rceil - 1. |S'_M| - 1$ covers the edges $4\left(\left(l - \left\lfloor\frac{l}{4}\right\rfloor\right) - 1\right) < \left\lceil\frac{q}{2}\right\rceil = 3l$.This set is not an inverse majority neighborhood set. Hence,$|S'_M| = n_i^{-1}(\chi) = \left(l - \left\lfloor\frac{l}{4}\right\rfloor\right) = \left\lceil\frac{3l}{4}\right\rceil$.

Theorem 2.4 Let $\chi = P_5 \otimes C_l$ be the graph with $l \geq 3$, then $n_i^{-1}(\chi) = l$.

Theorem 2.5. Let $\chi = P_6 \otimes C_l$ be the graph with $l \geq 3$, then $n_i^{-1}(\chi) = \left\lceil\frac{5l}{4}\right\rceil$.

Theorem 2.6. Let $\chi = P_7 \otimes C_l$ be the graph with $l \geq 3$, then $n_i^{-1}(\chi) = \left\lceil\frac{3l}{2}\right\rceil$.

Theorem 2.7. Let $\chi = P_8 \otimes C_l$ be the graph with $l \geq 3$, then $n_i^{-1}(\chi) = \left\lceil\frac{7l}{4}\right\rceil$.

Observation: Let $\chi = P_n \otimes C_l$ (Fig 2) be the graph with $n > 8, l \geq 3$, then $n_i^{-1}(\chi) = \left\lceil\frac{2(n-1)l}{4}\right\rceil$.

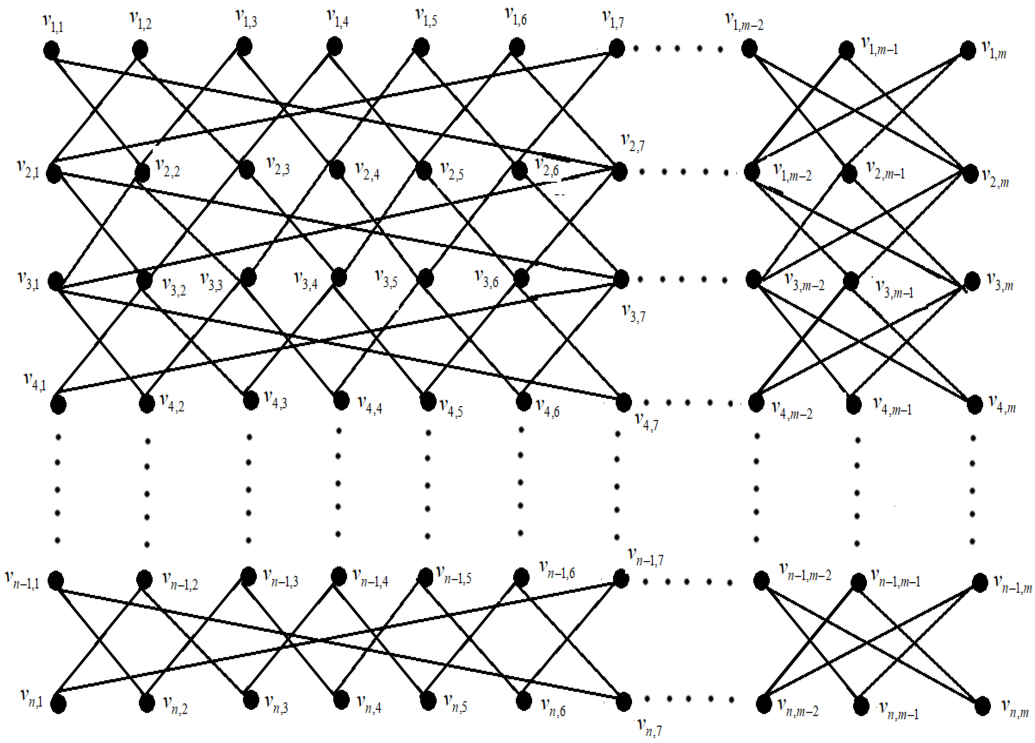

Figure 1: $G = P_n \otimes C_l$

The 'Figure 1' represents a generalised tensor product of path P_n and C_l in that vertices are mentioned$V(G) = \{v_{1,1}, v_{1,2}, v_{1,3} \dots v_{1,m}, v_{2,1}, v_{2,2}, \dots v_{2,m} \dots v_{n,1}, v_{n,2}, \dots v_{n,m}\}$

3 Majority neighborhood number of tensor product of path and cycle of Network Robustness and Connectivity

In communication networks, the concept of majority influence or connectivity could be key to understanding how messages or information propagate through the system. The tensor product of path and cycle graphs can model complex communication networks where multiple paths and cycles (such as redundant communication routes or multiple feedback loops) exist. By studying the majority neighborhood number in such a graph, network engineers can gain insights into how effectively information propagates across the network, and identify vulnerabilities or strengths in connectivity. A higher majority neighborhood number might suggest that communication between nodes is more resilient or faster, because nodes are more "closely connected" in terms of information flow. In the context of fault tolerance, the tensor product structure may be useful for modeling network recovery, as the combination of paths and cycles allows for rerouting communication in case of node or link failure.

Routing protocols in communication networks often seek to minimize latency and optimize resource allocation. The majority neighborhood number could provide a way to assess the efficiency of routing strategies, especially in networks where path diversity (via cycles) and linear ordering (via paths) play a role. In machine learning, clustering is often tied to feature selection and dimensionality reduction. The tensor product of a path and cycle can serve as a way to model relationships between features that have both ordered and cyclical properties.

The majority neighborhood number of a tensor product of a path and a cycle graph has direct relevance to both communication networks and data clustering because it provides a measure of connectivity and influence in complex structures. In communication networks, it could inform routing, fault tolerance, and distributed consensus. In data clustering, it could offer a means to better understand and model communities, relationships, and dependencies in data. By leveraging this mathematical concept, we can enhance the design and optimization of systems in fields, leading to more efficient, resilient, and accurate algorithms and architectures.

4 Conclusions

The main conclusion of studying the inverse majority neighborhood number for $P_n \times C_m$could include specific values or bounds of this number depending on n and m. Additionally, the results might reveal how structural properties of tensor products influence the inverse majority neighborhood characteristics, providing insight into efficient designs of networks with desired domination and independence traits. The future work around the majority neighborhood number in the tensor product of path and cycle graphs holds significant promise for both theoretical advancements and practical applications. From optimizing communication networks and improving data clustering algorithms to understanding dynamic systems in biological and social networks, this concept can offer new perspectives and tools for a wide range of domains. As researchers continue to explore these avenues, we can expect to see novel methodologies emerge that leverage the mathematical richness of graph theory for real-world problem-solving.

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5 Declarations

5.1 Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Unrelated & Excessive Self citation

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