Solution of Fuzzy Linear Equation Using Cramer's Rule

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ABSTRACT

Many real-time engineering systems are too complex to be defined in precise terms and imprecision is often involved. Linear system of equations with uncertainty parameters plays a significant role in the areas of economics, finance, engineering, control system, and so on. To analyze such situation, fuzzy information is required. In this paper deals in solving fuzzy system of linear equations. There are many non-fuzzy classical methods to solve linear equations but, in this paper, solving fuzzy linear equation and solving linear equation using Cramer's rule are discussed.

Keywords: Fuzzy set, fuzzy number, fuzzy linear equation, problem of finding an unknown number, solving linear equation using Cramer's rule, numerical example.

1 Introduction

System of linear equations are frequently found in various field viz. engineering science and economics etc. A general model for solving a fuzzy linear system of equations where coefficient matrix is crisp and the right-hand-side column is a fuzzy vector was first proposed by Friedman et al. [1], and later with his colleagues, they replaced the original fuzzy linear system by a crisp linear system and solved it. Then, Dehgan et al. [2] considered all parameters in a fuzzy linear system as fuzzy numbers and is called fully fuzzy linear system. The method is used in computing inverse of a matrix in fuzzy case that employs a linear equation system and identity matrix.

2 Preliminaries

Definition 2.1

A Fuzzy set is a set whose elements have degree of membership. Fuzzy sets are an extension of the classical notion of set. More mathematically, a fuzzy set is a pair(A,μ_A)where A is a set and μ_A:A⟶ [0,1]. For all x ε A, $\mu_A(x)$ is called the grade of membership of x.

Definition 2.2

A Fuzzy set A on R must possess at least the following three properties

⇒A must be a normal fuzzy set

⇒A_α must be a closed interval for every αε (0,1] (convex)

⇒the support of fuzzy set A,A_{0+}, must be bounded

Definition 2.3

A matrix equation is represented as$\tilde{A} \otimes \tilde{x}$=$\tilde{b}$,where $\tilde{A}$=$\widetilde{a_{ij}}$ is a fuzzy matrix $\tilde{A}$

of size (n x n) and $\tilde{b}$ and $\tilde{x}$ are fuzzy vectors of size (n x 1). If the vector or matrix elements are in intervals, then this system is called interval linear system.

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Definition 2.4

A matrix $\tilde{A} = (\tilde{a}_{ij})$is called a fuzzy matrix if each element of $\tilde{A}$ is a fuzzy number [3,4]. $\tilde{A}$ *is po*sitive if each element of $\tilde{A}$ >0 and is negative if each element of $\tilde{A}$<0.

Let$\tilde{A}$ ($\tilde{A} = (\tilde{a}_{ij})$) and $\tilde{B}$ ($\tilde{B} = (\tilde{b}_{ij})$) be two matrices of p x q and q x r. The size of the product of fuzzy matrices is p x r. The product is written as:

$$\tilde{A} \otimes \tilde{B} = \tilde{C}(\tilde{c}_{ij}),$$

Where $\otimes$ is approximated as multiplication. $\tilde{c}_{ij} = \sum_{k=1,2,3,\ldots n} \tilde{a}_{ik} \otimes \tilde{b}_{kj}$

A fuzzy matrix $\tilde{A}$,just like a fuzzy number, consists of center, left spread, and right spread and is represented as $\tilde{A}$= (A, L, R) where A, L, R are crisp matrices and these are denoted as center, left spread, and right spread, respectively.

A fuzzy number P is called LR-type fuzzy number if the membership function μ is of the following form [3]:

$$\mu_p = \begin{cases} L\left(\frac{p-x}{\alpha}\right), for\ x \leq p, \alpha > 0 \\ R\left(\frac{x-p}{\beta}\right), forx \geq p, \beta > 0 \end{cases}$$

Where L and Rare continuous decreasing function in the interval [0, ∞, +). It fulfills the condition L (0) =R (0) =1. p is the mean value of the fuzzy number P which is denoted as P= (p, α, β) whereα and β are left and right spread respectively, and these are positive real numbers [5,6].

3 Fuzzy Linear Equation

Consider a n x n linear system of linear equations:

$(\tilde{a}_{11} \otimes \tilde{x}_1) \oplus (\tilde{a}_{12} \otimes \tilde{x}_2) \oplus \ldots . \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) = \tilde{b}_1$

$(\tilde{a}_{21} \otimes \tilde{x}_1) \oplus (\tilde{a}_{22} \otimes \tilde{x}_2) \oplus \ldots . \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) = \tilde{b}_2$

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$(\tilde{a}_{n1} \otimes \tilde{x}_1) \oplus (\tilde{a}_{n2} \otimes \tilde{x}_2) \oplus \ldots . \oplus (\tilde{a}_{nn} \otimes \tilde{x}_n) = \tilde{b}_n$

The matrix form of above equation is written as:

$\tilde{A} \otimes \tilde{x} = \tilde{b}$ or $\tilde{A}\tilde{x} = \tilde{b}$

$\tilde{x}$ is a fuzzy approximate solution of $\tilde{A} \otimes \tilde{x} = \tilde{b}$ with left and right shape functions as L(.) and R(.)that are used to represent $\tilde{A}$ and $\tilde{b}$,where $\tilde{A} = [\tilde{a}_{ij}]$ is a coefficient matrix of n x n ,and the vector $\tilde{a}_{ij} = (\tilde{a}_{ij}, \tilde{\alpha}_{ij}, \tilde{\beta}_{ij})_{LR}$ and $\tilde{b}$=(b,m,n).

Let the unknown vector $\tilde{x}$ be represented as $\tilde{x}$= (X, Y, Z).

Considering x≥0, then we may write$\tilde{A} \otimes \tilde{x} = \tilde{b}$ as

(A, P, Q) ⊗ (X, Y, Z) = (b, m, n), where $\tilde{\alpha}_{ij}$=P, $\tilde{\beta}_{ij}$=Q is assumed.

Using the theory of multiplication of two fuzzy numbers,

For two positive fuzzy numbers P=$(p, \alpha, \beta)_{LR}$and Q=$(q, \gamma, \delta)_{LR}$

Multiplication:

$(p, \alpha, \beta)_{LR} \otimes (q, \gamma, \delta)_{LR} = (p.q, p\gamma + q\alpha, p\delta + q\beta)_{LR}$,P>0, Q>0

Then, (A, P, Q) ⊗ (X, Y, Z) = (AX, AY+PX, AZ+QX).

Thus, (AX, AY +PX, AZ+QX) = (b, m, n)

4 Problem of Finding an Unknown Number

Definition:4.1

Let $\tilde{x}$ is a fuzzy approximate solution of $\tilde{A} \otimes \tilde{x}$=$\tilde{b}$ with left and right shape functions similar to that of L(.) and R(.)functions which are used in $\tilde{A}$ and$\tilde{b}$ with approximate operator $\tilde{x}$ =(x,y,z)≥0,then $\tilde{x}$ is said to be a fuzzy solution of (A,P,Q)⊗ x̃=(b,m,n)if and only if:

AX=b, AY+PX=m, AZ+QX=n

The membership function of each element,μ_x>0 is defined from L and R functions that are used in$\tilde{A}$ and$\tilde{b}$

Assuming A to be a crisp matrix, then we can write

(AX, AY+PX, AZ+QX) = (b, m, n)

Thus, we have three equations

I)AX=b

II)AY+PX=m

⇒AY=m-PX

III)AZ+QX=n

⇒AZ=n-QX

So, we get

X=A^{-1}b

y=A^{-1}m-$A^{-1}Px$

z=A^{-1}n-$A^{-1}Qx$

Let us take an example to find the unknown vector $\tilde{x}$ of a fuzzy linear system, where the unknown vector is LR-type fuzzy numbers with left and right spread.

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5 Numerical Example

Consider a fuzzy linear system

$5\tilde{x}_1+6\tilde{x}_2=50$

$7\tilde{x}_1+4\tilde{x}_2=48$

Where x is a fuzzy approximate solution with y, z as the left and right spread functions. Compute $\tilde{x}=$ (x, y, z)

Solution:

Let the system in fuzzy form with left and right spread be written as:

(7,1,1) $\otimes$ $(x_1, y_1, z_1)\oplus$(4,0,1) $\otimes$(x_2, y_2, z_2)=(48,7,11)

From the two equations, we write the values of A, P, Q as

$$A=\begin{bmatrix}5 & 6\\7 & 4\end{bmatrix}, P=\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}, Q=\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}$$

In matrix form Ax=b, we can write:

$$\begin{bmatrix}5 & 6\\7 & 4\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}50\\48\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}5 & 6\\7 & 4\end{bmatrix}^{-1}\begin{bmatrix}50\\48\end{bmatrix}$$

On solving we get

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}=\frac{1}{-22}\begin{bmatrix}4 & -6\\-7 & 5\end{bmatrix}\begin{bmatrix}50\\48\end{bmatrix}$$

$$=\frac{1}{-22}\begin{bmatrix}200 & -288\\-350 & 240\end{bmatrix}$$

$$=\frac{1}{-22}\begin{bmatrix}-88\\-110\end{bmatrix}$$

$$=\begin{bmatrix}4\\5\end{bmatrix}$$

For obtaining $\begin{bmatrix}y_1\\y_2\end{bmatrix}$,we useAy=m-Px

We get $\begin{bmatrix}5 & 6\\7 & 4\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}12\\7\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}4\\5\end{bmatrix}$,

$$\begin{bmatrix}5 & 6\\7 & 4\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}12\\7\end{bmatrix}-\begin{bmatrix}9\\4\end{bmatrix}=\begin{bmatrix}3\\3\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}5 & 6\\7 & 4\end{bmatrix}^{-1}\begin{bmatrix}3\\3\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}y_1\\y_2\end{bmatrix}=\frac{1}{-22}\begin{bmatrix}4 & -6\\-7 & 5\end{bmatrix}\begin{bmatrix}3\\3\end{bmatrix}$$

$$=\frac{1}{-22}\begin{bmatrix}-6\\-6\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=\begin{bmatrix} \frac{3}{11} \\ \frac{3}{11} \end{bmatrix}$$

Likewise, for finding $\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$,we use, AZ=n-QX

$$\begin{bmatrix} 5 & 6 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 11 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 6 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}= \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 7 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$=\frac{1}{-22}\begin{bmatrix} 4 & -6 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}=\frac{1}{-22}\begin{bmatrix} 12 & -12 \\ -21 & 10 \end{bmatrix}=\begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}$$

Thus, the solution of the unknown vector $\tilde{x}$ is:

$$\tilde{x}=\begin{bmatrix} \left(4,\frac{3}{11},0\right) \\ \left(5,\frac{3}{11},\frac{1}{2}\right) \end{bmatrix}$$

6 Solving Linear Equation Using Cramer's Rule

Cramer's rule is a formula for the solution of a system of linear equation with as many equations as unknowns. The solution is expressed in terms of determinants of a square coefficient matrix and of matrices obtained from it by replacing one column of the coefficient matrix by the column vector of right-hand side of the equations.

We can write,$x_i=\frac{\det\left(A_1^i\right)}{\det(A)}$,$i$=1,2,3……n

Where A_1^i is obtained by replacing the ith column of A by b.

Then, using the value of x, AY=m-PX and AZ=n-QX are computed and the following values are obtained as:

$y_i=\frac{\det\left(A_2^i\right)}{\det(A)}$,$i$=1,2, 3……n

Where A_2^i is obtained by replacing ith column of A by m-Px

$z_i=\frac{\det\left(A_3^i\right)}{\det(A)}$,$i$=1,2,3……n

Where A_3^i is obtained by replacing ith column of A by n-Qx.

7 Numerical Example

Consider a fuzzy linear system

$4\tilde{x}_1+5\tilde{x}_2+3\tilde{x}_3=71$

$7\tilde{x}_1+10\tilde{x}_2+2\tilde{x}_3=118$

$6\tilde{x}_1+7\tilde{x}_2+15\tilde{x}_3=155$

Compute the vector $\tilde{x}$, $\tilde{x}$= (X, Y, Z) with y, z denoting the left and right spreads, respectively.

Solution:

Considering $\tilde{x}$= (X, Y, Z) to be a fuzzy approximate solution with left and right spread-shape functions, the fuzzy linear system may be written as:

$(4,1,0) \otimes (x_1, y_1, z_1) \oplus (5,3,2) \otimes (x_2, y_2, z_2) \oplus (3,0,3) \otimes (x_3, y_3, z_3)=(71,54,76)$

$(7,4,3) \otimes (x_1, y_1, z_1) \oplus (10,6,5) \otimes (x_2, y_2, z_2) \oplus (2,1,1) \otimes (x_3 y_3, z_3)=(118,113,129)$

$(6,2,4) \otimes (x_1, y_1, z_1) \oplus (7,1,2) \otimes (x_2, y_2, z_2) \oplus (15,5,4) \otimes (x_3 y_3, z_3)=(155,89,151)$

In matrix form, the above equations may be written as:

$$\begin{bmatrix}(4,1,0) & (5,3,2) & (3,0,3)\\(7,4,3) & (10,6,5) & (2,1,1)\\(6,2,4) & (7,1,2) & (15,5,4)\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}(71,54,76)\\(118,113,129)\\(155,89,151)\end{bmatrix}$$

Here, $A=\begin{bmatrix}4 & 5 & 3\\7 & 10 & 2\\6 & 7 & 15\end{bmatrix}$, $p=\begin{bmatrix}1 & 3 & 0\\4 & 6 & 1\\2 & 1 & 5\end{bmatrix}$, $Q=\begin{bmatrix}0 & 2 & 3\\3 & 5 & 1\\4 & 2 & 4\end{bmatrix}$

First, we will solve for x using the equation: Ax=b

$$A=\begin{bmatrix}4 & 5 & 3\\7 & 10 & 2\\6 & 7 & 15\end{bmatrix}$$

det A=4(150-14)-5(105-12) +3(49-60)

=544-465-33=46

det$A_1^{(1)}$is obtained by replacing the first column of matrix Aby b.so,

$$A_1^{(1)}=\begin{bmatrix}71 & 5 & 3\\118 & 10 & 2\\155 & 7 & 15\end{bmatrix}$$

=71(150-14)-5(1770-310) +3(826-1550)

det$A_1^{(1)}$=9656-7300-2172=184

Similarly,

$$A_1^{(2)}=\begin{bmatrix}4 & 71 & 3\\7 & 118 & 2\\6 & 155 & 15\end{bmatrix}$$

det$A_1^{(2)}$ =5840-6603+1131=368

$$A_1^{(3)}=\begin{bmatrix}4 & 5 & 71\\ 7 & 10 & 118\\ 6 & 7 & 155\end{bmatrix}$$

det$A_1^{(3)}$ = 2896-1885-781=230

We get,

$x_1=\frac{detA_1^{(1)}}{detA}=\frac{184}{46}=4;$

$x_2=\frac{detA_1^{(2)}}{detA}=\frac{368}{46}=8;$

$x_3=\frac{detA_1^{(3)}}{detA}=\frac{230}{46}=5.$

So, $\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}4\\ 8\\ 5\end{bmatrix}$

Likewise, we compute for y and z, we have Ay=m-PX

$$=\begin{bmatrix}54\\ 113\\ 89\end{bmatrix}-\begin{bmatrix}1 & 3 & 0\\ 4 & 6 & 1\\ 2 & 1 & 5\end{bmatrix}\begin{bmatrix}4\\ 8\\ 5\end{bmatrix}$$

$$=\begin{bmatrix}54\\ 113\\ 89\end{bmatrix}-\begin{bmatrix}28\\ 69\\ 41\end{bmatrix}=\begin{bmatrix}26\\ 44\\ 48\end{bmatrix}$$

$$y=\begin{bmatrix}4 & 5 & 3\\ 7 & 10 & 2\\ 6 & 7 & 15\end{bmatrix}^{-1}\begin{bmatrix}26\\ 44\\ 48\end{bmatrix}$$

Proceeding a similar manner as above, we obtain$A_2^{(1)}$, $A_2^{(2)}$, $A_2^{(3)}$ by replacing first, second, and third column of matrix A by(m-Px), respectively, we get

det$A_2^{(1)}$=$\begin{bmatrix}26 & 5 & 3\\ 44 & 10 & 2\\ 48 & 7 & 15\end{bmatrix}$=26(150-14)-5(660-96) +3(308-480) =200

det$A_2^{(2)}$=$\begin{bmatrix}4 & 26 & 3\\ 7 & 44 & 2\\ 6 & 48 & 15\end{bmatrix}$=4(660-96)-26(105-12) +3(336-264) =54

det$A_2^{(3)}$=$\begin{bmatrix}4 & 5 & 26\\ 7 & 10 & 44\\ 6 & 7 & 48\end{bmatrix}$=4(480-308)-5(336-264) +26(49-60) =42

we get

$y_1=\frac{detA_2^{(1)}}{detA}=\frac{200}{46}=\frac{100}{3};$

$y_2=\frac{detA_2^{(2)}}{detA}=\frac{54}{46}=\frac{27}{23};$

$y_3 = \frac{detA_2^{(3)}}{detA} = \frac{42}{46} = \frac{21}{23}$.

So, $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \frac{100}{23} \\ \frac{27}{23} \\ \frac{21}{23} \end{bmatrix}$

Likewise, for z, Az = n-QX

$$Az = \begin{bmatrix} 76 \\ 129 \\ 151 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 3 \\ 3 & 5 & 1 \\ 4 & 2 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 5 \end{bmatrix} = \begin{bmatrix} 76 \\ 129 \\ 151 \end{bmatrix} - \begin{bmatrix} 31 \\ 57 \\ 52 \end{bmatrix} = \begin{bmatrix} 45 \\ 72 \\ 99 \end{bmatrix}$$

$$Z = \begin{bmatrix} 4 & 5 & 3 \\ 7 & 10 & 2 \\ 6 & 7 & 15 \end{bmatrix}^{-1} \begin{bmatrix} 45 \\ 72 \\ 99 \end{bmatrix}$$

Proceeding in a similar manner as above we obtain $A_3^{(1)}$, $A_3^{(2)}$, $A_3^{(3)}$by replacing the first, second and third columns of matrix A by (n-Qx), respectively, we get

$$detA_3^{(1)} = \begin{bmatrix} 45 & 5 & 3 \\ 72 & 10 & 2 \\ 99 & 7 & 15 \end{bmatrix} = 45(150-14)-5(1080-198) +3(504-990) = 252;$$

$$detA_3^{(2)} = \begin{bmatrix} 4 & 45 & 3 \\ 7 & 72 & 2 \\ 6 & 99 & 15 \end{bmatrix} = 4(1080-198)-45(105-12) +3(693-432) = 126;$$

$$detA_3^{(3)} = \begin{bmatrix} 4 & 5 & 45 \\ 7 & 10 & 72 \\ 6 & 7 & 99 \end{bmatrix} = 4(990-504)-5(693-432) +45(49-60) = 144$$

we get

$z_1 = \frac{detA_3^{(1)}}{detA} = \frac{252}{46} = \frac{126}{23}$;

$z_2 = \frac{detA_3^{(2)}}{detA} = \frac{126}{46} = \frac{63}{23}$;

$z_3 = \frac{detA_3^{(3)}}{detA} = \frac{144}{46} = \frac{72}{23}$.

So, $\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} \frac{126}{23} \\ \frac{63}{23} \\ \frac{72}{23} \end{bmatrix}$

So, the fuzzy solution of $\tilde{x}$ is:

$$\tilde{x} = \begin{bmatrix} (4, \frac{100}{23}, \frac{126}{23}) \\ (8, \frac{27}{23}, \frac{63}{23}) \\ (5, \frac{21}{23}, \frac{72}{23}) \end{bmatrix}$$

8 Conclusion

In this paper, we have presented a method for fuzzy linear systems with fuzzy coefficients that involves fuzzy variables. Fuzzy linear equation has been solved using both direct method and Cramer's rule method. It is useful in analysis of physical systems and other topics in real engineering problems, where uncertainty aspects are present. For example, in finite elements for heat-transfer problems or finite-element formulation of equilibrium and steady-state problems in other areas, solving a set of simultaneous algebraic linear equations is required.

9 Declarations

9.1 Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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