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Geodetic Convexity in the Heisenberg Group

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ABSTRACT

A Heisenberg group $\mathbb{H}^n$(where $n \geq 1$) is a Lie group $\mathbb{C}^n \times \mathbb{R} = \{(z,t) | z \in \mathbb{C}^n, t \in \mathbb{R}\}$ together with the group operation defined as $(z,t)(w,s) = \left(z + w, t + s + 2\mathrm{Im}(z.\overline{w})\right)$ where $z.\overline{w} = \sum_{j=1}^{n} z_j \overline{w_j}$ is the Hermitian inner product in the complex space. This group structure imposes constraints on motions in the space $\mathbb{H}^n$ giving rise to a geometry which is sub-Riemannian but not Riemannian. Various notions of convex sets have been studied in the Heisenberg Group $\mathbb{H}^n$ (for $n \geq 1$) which may not necessarily be equivalent. A few of them include horizontal convexity, group convexity, convex in the viscosity sense and geodetic convexity. Here, we discuss the concept of geodetically convex sets in $\mathbb{H}^n$ for $n \geq 1$ and classify them. A geodetically convex set in $\mathbb{H}^n$ is defined to be a set which contains every geodesic connecting every pair of points in the set. We prove that every geodetically convex set in $\mathbb{H}^n$ is either an empty set, a singleton set, an arc of a geodesic or the whole space $\mathbb{H}^n$. These results generalise the known results of $\mathbb{H}^1$ to $\mathbb{H}^n$ for $n \geq 1$.

Keywords: Heisenberg; geodetic; horizontal

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