

# Fractional Order Observer-Based Adaptive Synchronization for a Class of Chaotic Systems with Unknown Parameters

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## ABSTRACT

In this research work, the problem of synchronizing two continuous-time Chua's chaotic systems with transmitter–receiver configuration is considered. Based on Lyapunov theory and fractional order properties, a fractional order adaptive observer-based response system is designed to achieve the synchronization of the two systems and to estimate the transmitter's unknown parameters. Simulation results illustrate the efficiency of the proposed adaptive synchronization scheme.

**Keywords:** Fractional order Chua's circuit, Chaos synchronization, Adaptive observer

## 1 Introduction

Recently, much work has focused on the introduction of fractional operators in the modeling, control and synchronization of chaotic systems. In particular, the adaptive approach has gained a lot of followers and more and more publications are emerging in the best scientific platforms [1]- [3].

This work motivation comes from its practical applications in a number of engineering domains such as image encryption, secure communication and so one [4]. In particular, the synchronization has been considered as a design of observer. Most design strategies made the assumption that all system characteristics were known and states could be measured. Nowadays, more and more research works focus on the synchronization of uncertain systems [5]- [7].

This study concerns the development of an adaptive observer for chaotic systems. Based on Lyapunov function design for the error system and fractional order calculus theory, a fractional order observer is designed for estimating unknown parameters of the driver system.

## 2 Problem Setting

We are interested by a transmitter with unknown parameters presenting chaotic behavior that can be represented by the following model:

$$\begin{cases} \dot{x} = Ax + \varphi_0(y) + B \sum_{i=1}^m \theta_i \varphi_i \\ y = Cx \end{cases} \quad (1)$$

where  $x$  is the transmitter state vector,  $y$  is the outputs vector.  $\theta$  is the unknown parameters vector.

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + \varphi_0(\hat{y}) + B \sum_{i=1}^m \hat{\theta}_i \varphi_i(\hat{y}) + L(y - C\hat{x}) \\ y = C\hat{x} \end{cases} \quad (2)$$

$$D^\alpha \hat{\theta} = \gamma \varphi(x)^T H (y - C\hat{x}) \quad (3)$$

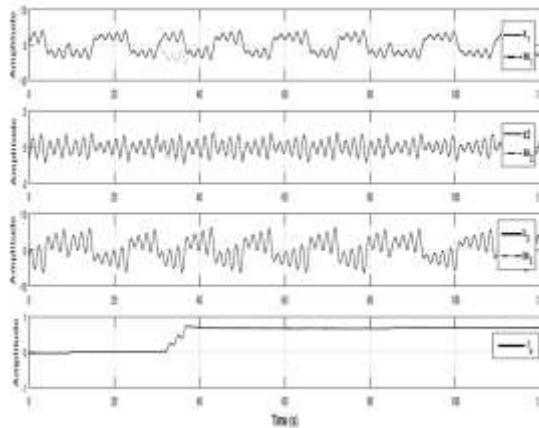
$$\lim_{t \rightarrow \infty} [\theta - \hat{\theta}] = 0 \quad (4)$$

## 3 Main results

**Theorem 1.** Assume that all the trajectories of the drive system (1) are bounded and it satisfies hypothesis 1 and 2: Then all the trajectories of the observer-based response system (2) associated with the fractional



order adaptation law (3) globally asymptotically synchronizes with the drive system. If, in addition, the vector-function  $\varphi(t)$  satisfies the PE condition, then also (4) holds.



**Figure 1:** Fractional order adaptive observer-based synchronization of Chua's circuit.

## 4 Conclusion

In this work, we have proposed a new fractional order adaptive synchronization system design based on an observer for a class of uncertain Chua's chaotic systems. The stability analysis of this algorithm was performed based on Lyapunov theory and fractional order systems' properties. The proposed observer-based adaptive synchronization scheme with a fractional order adaptation law demonstrates good reconstruction accuracy of the unknown master system parameters.

## 5 Competing Interests

The authors declared that no conflict of interest exists in this work.

## How to Cite

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