Free Vibration Analysis of Skew Sandwich Plate using Radial Basis Collocation Method

Jigyasa Singh*, Ram Bilas Prasad

Mechanical Engineering Department, Madan Mohan Malviya University of Technology, Gorakhpur, India
*Corresponding author's e-mail: jigyasasingh0612@gmail.com
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ABSTRACT

In this study, the free vibration analysis of a skew sandwich plate is conducted utilizing the Radial Basis Collocation Method (RBCM). Using HSDT, the free vibration response of a skew sandwich plate has been determined. Hamilton's approach is used to get the GDEs, which are then discretized using the RBF. The accuracy and efficiency of the RBCM in predicting the vibration and mode shapes of the skew sandwich plate are demonstrated through a comparison with results obtained from existing analytical and numerical methods. The impact of various parameters on the dynamic response of the plate is analyzed, providing valuable insights into the design and optimization of skew sandwich structures. Findings from the open literature are used to validate the current findings. Examined are the effects of the skew angle, core-to-face thickness ratio, and span-to-thickness ratio on the frequency.

Keywords: Skew plate, HSDT, Sandwich, Meshfree, Vibration

1 Introduction

Due to its beneficial qualities, for example very high stiffness-to-weight and strength-to-weight ratios, thermal characteristics, and several other multi-physical attributes, laminated composites, and sandwich configurations are used in many lightweight structures in aviation, structural, maritime, and civil engineering applications. By giving designers of mechanical parts the flexibility to customize the distribution of materials of various qualities following the loading routes, these designs also enable custom optimization. Laminated composites are constructed by layering piles of composites with specific fibre orientations in each layer to create the structures. However, the vibration analysis of skew laminated sandwich plates hasn't received much attention in the literature. Shi *et al.* [1] used a semi-analytical method to analyse how sandwich plates buckled. The finite element approach was used by Karakoti *et al.* [2] to examine skew-edge sandwich plates. Katariya *et al.* [3] studied statics and natural frequency of skew sandwich composite plates using the HSDT model.

2 Mathematical Formulation

Figure 1 depicts a skew plate's geometry where thickness 'h' over the z-axis has been considered.

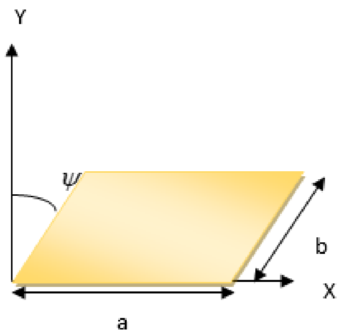

Figure 1: *Skew plate's geometry*

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The displacement variable is expressed by Singh *et al.* [4]:

$$u=u_0-z\frac{\partial w_0}{\partial x}+f(z)\phi_x,\quad v=v_0-z\frac{\partial w_0}{\partial y}+f(z)\phi_y,\quad w=w_0 \tag{1}$$

For the present analysis $f(z)=z\,e^{-2\left(z/h\right)^2}$ have been considered as proposed by Karama [5].

The GDEs of the skew plate are derived using the Hamilton principle, which is written as:

$$\delta u_0:\quad \frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=\left(I_0\frac{\partial^2 u_0}{\partial \tau^2}-I_1\frac{\partial^3 w_0}{\partial x\partial \tau^2}+I_3\frac{\partial^2 \phi_x}{\partial \tau^2}\right) \tag{2}$$

$$\delta v_0:\quad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=\left(I_0\frac{\partial^2 v_0}{\partial \tau^2}-I_1\frac{\partial^3 w_0}{\partial y\partial \tau^2}+I_3\frac{\partial^2 \phi_y}{\partial \tau^2}\right) \tag{3}$$

$$\delta w_0:\quad \frac{\partial^2 M_{xx}}{\partial x^2}+\frac{\partial^2 M_{yy}}{\partial y^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}=$$

$$I_0\frac{\partial^2 w_0}{\partial t^2}+I_1\left(\frac{\partial^3 u_0}{\partial x\partial \tau^2}+\frac{\partial^3 v_0}{\partial y\partial \tau^2}\right)-I_2\left(\frac{\partial^4 w_0}{\partial x^2\partial \tau^2}+\frac{\partial^4 w_0}{\partial y^2\partial \tau^2}\right)+I_4\left(\frac{\partial^3 \phi_x}{\partial x\partial \tau^2}+\frac{\partial^3 \phi_y}{\partial y\partial \tau^2}\right) \tag{4}$$

$$\delta \phi_x:\quad \frac{\partial M^f_{xx}}{\partial x}+\frac{\partial M^f_{xy}}{\partial y}-Q^f_x=\left(I_3\frac{\partial^2 u_0}{\partial \tau^2}-I_4\frac{\partial^3 w_0}{\partial x\partial \tau^2}+I_5\frac{\partial^2 \phi_x}{\partial \tau^2}\right) \tag{5}$$

$$\delta \phi_y:\quad \frac{\partial M^f_{xy}}{\partial x}+\frac{\partial M^f_{yy}}{\partial y}-Q^f_y=\left(I_3\frac{\partial^2 v_0}{\partial \tau^2}-I_4\frac{\partial^3 w_0}{\partial y\partial \tau^2}+I_5\frac{\partial^2 \phi_y}{\partial \tau^2}\right) \tag{6}$$

where

$$N_{ij},\, M_{ij},\, M^f_{ij}=\int_{-h/2}^{+h/2}\left(\sigma_{ij},\, z\sigma_{ij},\, f(z)\sigma_{ij}\right)dz \tag{7}$$

$$Q^f_x,\, Q^f_y=\int_{-h/2}^{+h/2}\left(\sigma_{xz},\, \sigma_{yz}\right)\left(\frac{\partial f(z)}{\partial z}\right)dz \tag{8}$$

The simply supported boundary conditions are taken as:

$$u_s,\phi_s,w_0,N_{nn},M_{nn}=0 \tag{9}$$

Where,

$$u_s=-n_y.u_o+n_x.v_0$$
$$\phi_s=-n_y.\phi_x+n_x.\phi_y$$
$$N_{nn}=n_x^2N_{xx}+2n_xn_yN_{xy}+n_y^2N_{yy}$$
$$M_{nn}=n_x^2M_{xx}+2n_xn_yM_{xy}+n_y^2M_{yy}$$
$$n_x=\cos(\psi),\quad n_y=\sin(\psi) \tag{10}$$

3 Solution Methodology

Figure 2 depicts a skew domain with NB boundary nodes and NI inner nodes.

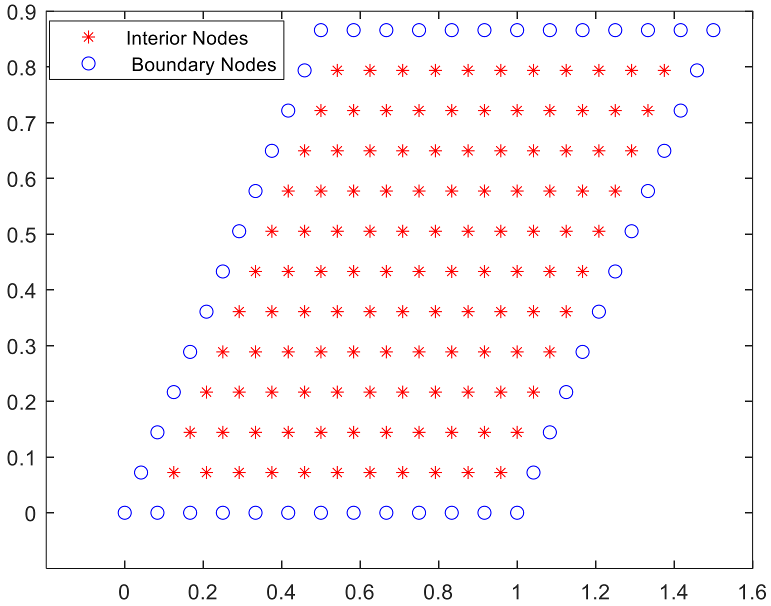

Figure 2: *Geometry of skew plate with nodes*

The field variables u_0, v_0, w_0, ϕ_x and ϕ_y of Eq (1) are assumed in terms of radial basis function as:

$$u_0 = \sum_{j=1}^{N} \alpha_j^{u_0} g\left(\left\|X - X_j\right\|, c\right),\ v_0 = \sum_{j=1}^{N} \alpha_j^{v_0} g\left(\left\|X - X_j\right\|, c\right),\ w_0 = \sum_{j=1}^{N} \alpha_j^{w_0} g\left(\left\|X - X_j\right\|, c\right)$$
$$\phi_x = \sum_{j=1}^{N} \alpha_j^{\phi_x} g\left(\left\|X - X_j\right\|, c\right),\ \phi_y = \sum_{j=1}^{N} \alpha_j^{\phi_y} g\left(\left\|X - X_j\right\|, c\right) \tag{11}$$

Where, $[\alpha^{u_0}\ \alpha^{v_0}\ \alpha^{w_0}\ \ \alpha^{\phi_x}\ \ \alpha^{\phi_y}]$ are unknown coefficients, $g\left(\left\|X - X_j\right\|, c\right)$ is RBF, $\left\|\mathrm{X} - \mathrm{X}_j\right\|$ is the form parameter and is the radial separation between the nodes. RBF chosen for the analysis at hand is $g = r^{2c-1}$

Where $r = \left\|X - X_j\right\| = \sqrt{\left((x - x_j)/a\right)^2 + \left((y - y_j)/b\right)^2}$ (12)

Using Eqn (11), the GDEs, along with boundary conditions, and represented as compact matrices that have been discretized as:

$$[\mathrm{K}]\{\delta\} = [\mathrm{M}]\{\delta\} \tag{13}$$

$$\{\delta\} = [\alpha^{u_0}\ \alpha^{v_0}\ \alpha^{w_0}\ \ \alpha^{\phi_x}\ \ \alpha^{\phi_y}]^T \tag{14}$$

Which may be expressed as:

$$\left[\begin{bmatrix}[K]_L \\ [K]_B\end{bmatrix}_{5N\times 5N} + \omega^2 \begin{bmatrix}[M] \\ 0\end{bmatrix}_{5N\times 5N}\right]\{\delta\}_{5N\times 1} = 0 \tag{15}$$

The eigenvectors (*V*) and eigenvalues (*D*) are calculated as:

$$[V, D] = eig\left[\begin{bmatrix}[K]_L \\ [K]_B\end{bmatrix}_{5N\times 5N}, \begin{bmatrix}[M] \\ 0\end{bmatrix}_{5N\times 5N}\right] \tag{16}$$

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Frequency (ω) = $\sqrt{D}$

4 Results and Discussion

Present area focuses on numerical experimentations and validation of obtained results. Considered is a square sandwich plate (a/h=10) with a core layer that is 0.8 times thick and two orthotropic face layers that are each 0.1 times thick. The following material characteristics were used for the core layer:

E_2=0.543, E_1=1, ν_{12}=0.3, G_{12}=0.2629, G_{13}=0.1599, G_{23}=0.2668; ρ =1.

The elastic modulus of face sheets has been varied with a factor R_f.

An analysis of the fundamental frequency validity and parameter's convergence $\Omega = 100\omega\left(\frac{\rho h^2}{E_1}\right)^{(1/2)}$ is presented in **Error! Reference source not found.**. Results obtained are compared with results due to S rinivas and Rao [6]. It can be seen that the present results converged within 1% and became closer to the results of Srinivas and Rao [6] at 15X 15 nodes.

Table 1: *Frequency parameter* Ω *for different* R_f.

Method	R_f					
	1	2	5	10	15	20
Present(5x5)	3.0387	3.3288	4.0769	5.0848	5.9236	6.6576
Present(7x7)	4.7366	5.7041	7.7592	9.9958	11.5890	12.8522
Present(9x9)	4.7397	5.7059	7.7511	9.9586	11.5150	12.7378
Present(11x11)	4.7322	5.6961	7.7334	9.9255	11.4646	12.6692
Present(13x13)	4.7274	5.6899	7.7230	9.9075	11.4383	12.6342
Present(15x15)	4.7246	5.6863	7.7171	9.8976	11.4242	12.6156
Srinivas and Rao [6]	4.7419	5.7041	7.7148	9.8104	11.2034	------

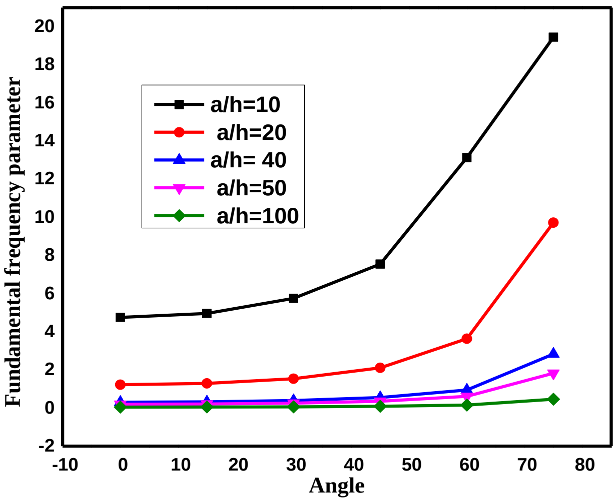

Figure 3: *Influence of skew angle and span-to-thickness ratio on* Ω

Figure 3 illustrates how the span-to-thickness ratio and skew angle affect. It has been seen that the skew angle starts to rise with increasing value. It is also noted that the fundamental frequency parameter lowers as the as the ratio of span to thickness rises, this effect is negligible after a/h= 50.

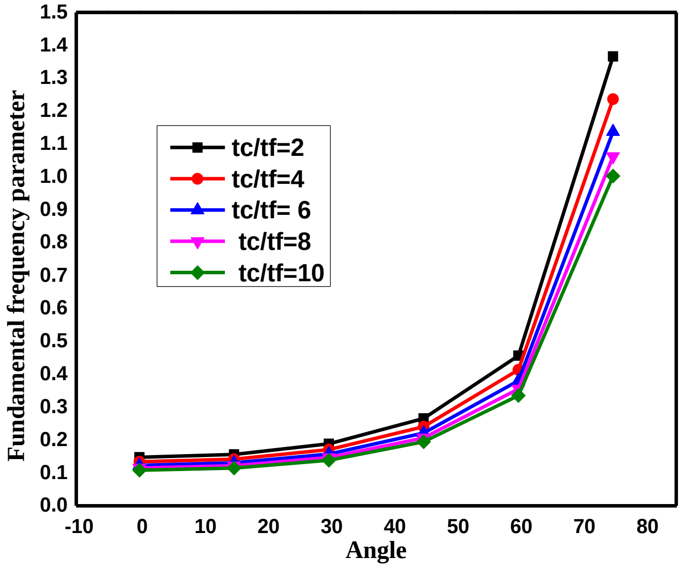

Figure 4: *Influence of skew angle and core to face thickness ratio on* Ω

Impact of the skew angle and core-to-face thickness ratio is seen in Figure 4. It is evident that starts growing as the value of the skew angle increases. The fundamental frequency parameter diminishes as the value of tc/tf increases, and when tc/tf=8, the influence of tc/tf is minimal. Figure 5 shows the contours of the first three frequencies of the skew sandwich plate. It is clearly observed that the present method and present displacement model is capable of obtaining the shape of the contour for three modes of frequency.

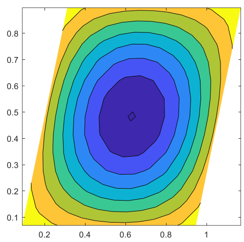	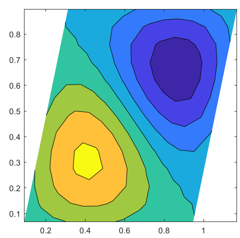	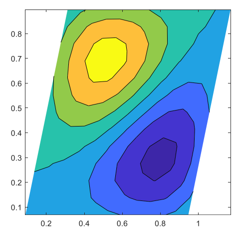
Mode-1	Mode-2	Mode-3

Figure 5: *Different mode frequency of sandwich plate [Rf=15, hc/hf=8,* ψ *=*15^0*]*

5 Conclusions

- The present solution methodology is easy to implement for good results.
- The frequency parameter rises as the skew angle increases.
- The frequency parameter diminishes as the span to thickness ratio rises, and beyond a/h =50, its impact is minimal.
- The frequency parameter reduces as the core-to-face thickness ratio rises.

6 Declarations

6.1 Competing Interests

The authors declare that they have no competing interests related to the research presented in this paper.

6.2 Publisher's Note

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How to Cite

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