# Review of Analytical Methods and Recent Advancement in Slope Stability Analyses

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### ABSTRACT

This paper covers two key aspects concerning slope analysis and design. In the first part, different analytical methods are reviewed and a method of limit equilibrium slope analysis that allows the interslice force inclinations to vary is presented. The new approach (referred to as the Arup Method), applicable on both circular and non-circular slips, is a further refinement on the popular Bishop and Janbu methods and is designed to overcome the numerical difficulties stemming from interlock. The proposed approach achieves overall horizontal, vertical and moment equilibrium of the slope, while also keeping every slice in horizontal and vertical equilibrium. Illustrative examples are presented to compare results from this method against recognized methods of analysis, including Morgenstern-Price, which employs a user-defined interslice force function. In the second part of the paper, development of a digitalised workflow for slope analyses and design is discussed and the authors demonstrate how customised coding enables optimisation of slope design involving soil nailing.

Keywords: Slope stability, Automation, Soil nailing

#### 1 Introduction

Current geotechnical practice widely adopts the method of slices, such as Bishop (1955), Janbu (1957), Spencer (1967) and Morgenstern-Price (1965), to analyse slope stability using limit equilibrium. These methods divide the slipping mass of slope into several vertical slices, onto which equations of static equilibrium are subsequently applied. They differ fundamentally based on the conditions of equilibrium they satisfy, and their assumptions regarding the interslice forces involved.

While the discussion on the merits and deficiencies of those methods have been made in previous publications such as GEO Report No. 208 (GEO, 2007), it comes to the attention of the authors that misunderstanding have been developed over the years among some practitioners, in particular to some of the variations of the rigorous forms of the popular Bishop and Janbu methods. This paper will propose the use of variably inclined interslice forces that Arup has developed in Bishop and Janbu methods. This method will from hereon be referred to as the Arup method and is a refinement of the Bishop and Janbu methods for slope analysis. The mechanics behind the Arup method is subsequently explained and case examples are used to compare results using this method against other analytical approaches, including Morgenstern & Price, which employs a user-defined interslice force function instead. Implications of using this approach for the different slope scenarios are then discussed.

On the other hand, with the recent advancements in computation, streamlining of the analysis process and optimisation through rapid parametric study is also possible. The second part of this paper introduces the development of a digitalised workflow for slope analyses and describe how customised coding enables optimisation of slope design adopting soil nailing.



## 2 Limit Equilibrium Analyses for Slope Stability

In the popular method of slices using limit equilibrium, a slip surface of some geometric shape (such as a wedge, circular or log spiral) is assumed, and the slipping mass is divided into a series of vertical slices, as shown in Figure 1. Various equations of static equilibrium are then applied either to each slice or to the slipping mass as a whole.



Figure 1: Method of slices in slope stability analysis

Analysing slope stability is, however, an indeterminate problem since there are more unknowns than equations. Sufficient assumptions are, hence, required to make the problem statically determinate.

Various analytical methods have been developed and adopted to evaluate slope stability. Some of the successfully adopted methods include Bishop, Janbu, Morgenstern & Price and Spencer. All these methods differ in the conditions of equilibrium they satisfy and the assumptions they make to render the problem statically determinate. In both Bishop and Janbu methods, horizontal inter-slice force is assumed. However, the two methods differ in the conditions of equilibrium they satisfy. The Bishop method satisfies vertical force equilibrium for each slice and overall moment equilibrium, while the Janbu method satisfies vertical force equilibrium for each slice, as well as overall horizontal force equilibrium for the entire slip mass. In the Morgenstern-Price method, the inter-slice shear forces are assumed to be related to the inter-slice normal forces using an arbitrary function. The user-defined function allows the inclinations of the interslice forces to vary between successive slices, and the Morgenstern-Price method can satisfy all conditions of equilibrium for the slipping mass as well as for each slice. Despite being more complex, this method is regarded as the most accurate method of slope stability analysis and is used most abundantly. The Spencer method, on the other hand, assumes a constant for the interslice function, resulting in parallel inclined interslice forces. The Spencer method is able to satisfy all conditions of overall equilibrium for the slipping mass.

Several studies and publications have investigated the merits and deficiencies of the different limit equilibrium methods (GEO, 2007; Wright *et al.*, 1973). According to these studies, there are several fundamental limitations of the different methods, especially the simpler methods of Bishop and Janbu methods. One of these limitations is the inability of some analytical methods (like Bishop and Janbu) to satisfy all the conditions of equilibrium. While this is true, several studies (Chirapuntu & Duncan, 1976; Whitman & Bailey, 1967) have observed that despite not meeting all conditions of equilibrium, the errors in the calculated factors of safety from these simpler methods (for example, Bishop) are usually small and on the safe side. According to Chirapuntu & Duncan (1976), the Bishop method gives result well within about 5% of the results calculated by the more accurate methods that achieve all

equilibrium conditions, even for effective stress analyses with high pore pressures. However, there are situations in which these relatively simple analytical methods may encounter issues. Whitman & Bailey (1967) investigated the formulation of Bishop method and its implication in various slope scenarios. They noted that the Bishop method may incur numerical difficulties and provide misleading answers, in special instances involving interlock.

#### 2.1 Interlock and numerical errors in the Bishop method

Despite being largely useful and accurate, the Bishop method incurs numerical problems and may potentially give unreliable results when interlock is involved. The interlock arises in the case of a deep slip with a low factor of safety, where the toe of the slip surface passes steeply through a highly cohesive and/or dense granular material (Chirapuntu & Duncan, 1976; Whitman & Bailey, 1967).

Oasys Ltd. (2022) explains this interlock phenomenon. If a deep slip emerges at a steep angle and has a high cohesion or mobilized angle of friction, then the direction of the resultant force, R, on the base of the slice may be almost horizontal or even pointing downwards. This can be visualized when referring to Figure 2.



Figure 2: Interlock in slope stability analysis, shown a) on section and b) using force diagram (Oasys Ltd., 2022)

To satisfy equilibrium of this slice, the interslice force, X, must point upwards. This direction is not consistent with the assumption of either horizontal (i.e., Bishop and Janbu methods) or even parallel inclined interslice forces (i.e., Spencer method). In such cases, the inclination of the interslice forces should be allowed to vary to act in the correct direction and take interlock into account, as shown in Figure 3.



Figure 3: Inclination of interslice forces for different slope methods (Oasys Ltd., 2022)

Whitman & Bailey (1967) explained the source of the difficulty, using the expression for effective normal force at the base of the slice that is adopted in the method ( $\overline{N}$ ), as shown in Equation (1).

$$\overline{N} = \frac{W - u\Delta x - c'\,\Delta x\,\tan\alpha/F}{\cos\alpha + \tan\theta'\sin\alpha/F} \tag{1}$$

where W = weight of the slice, u = pore pressure,  $\Delta x$  = length of the base of the slice,  $\alpha$  = inclination of the base of the slice with horizontal, c' = effective cohesion,  $\emptyset'$ = effective friction angle, and F = factor of safety

Referring to the above expression for  $\overline{N}$ , it can be observed that the denominator can possibly take on either very small or negative values towards the outward toe end of a slip circle where the angle of base inclination ( $\alpha$ ) is negative and of large magnitude. If this denominator is small, the value of  $\overline{N}$  will be very large and so will the frictional component of the strength for this slice. If the denominator is negative, the normal force and frictional component of strength will also be negative. In both circumstances the Bishop method would give unreasonable values of safety factors.

Similarly, for slopes in cohesive soils, another similar numerical difficulty may arise, where the numerator in the above expression may take on a negative value if cohesion is very high. This may result in the effective normal forces on the base of the slices of the slope to be negative, which would not be reasonable, and the calculated factor of safety would be unreasonably low.

## 3 Slope Stability Equations with Variable Interslice Force Inclinations – The Arup Method

The inclination of the interslice forces is a key consideration for an accurate slope stability analysis, and the assumptions made for the different methods of analysis will have impact on how the forces are calculated and how the conditions of equilibrium between slices met. To overcome the problem of interlock, the analytical methods would need to allow the inclinations of the interslice forces to vary and follow the right direction.

## 3.1 The Arup method

In this method developed by Arup (Oasys Ltd., 2022), Bishop and Janbu methods are refined further by enabling the inclinations of the interslice forces to vary. Under this approach, the program calculates the interslice forces to maintain horizontal and vertical equilibrium of each slice first. The inclinations of the interslice forces are then varied in each iteration until overall horizontal, vertical and moment equilibrium is also achieved.

This method is currently available in Oasys Slope program (Oasys Ltd., 2022) as an option to choose when running Bishop or Janbu Methods for slope stability analysis. Oasys Slope is Arup's software program (Oasys Ltd., 2022) that is used for slope stability analysis using limit equilibrium. The program offers a variety of widely used methods for performing slope stability checks, including Ordinary (or Swedish circle (Fellenius) method), Bishop, Janbu, Spencer and Morgenstern & Price methods.

Since the program uses iteration to reach convergence, the interslice force is adjusted separately, for both the vertical and horizontal directions by adding the fraction of the residual values from the previous iteration. The fraction is determined by the horizontal length of the slip surface represented by that slice. The interslice force direction is, hence, free to vary through this approach, but each slice is always in equilibrium along with the slipping mass, as a whole.

The variably inclined interslice force option is available on both circular and non-circular slip surfaces, and is a useful function designed to overcome the problem of interlock when using analytical methods that restrict the direction of the interslice forces.

The variably inclined method is preferable to horizontal and even parallel inclined interslice force methods, as it keeps every slice in horizontal and vertical equilibrium at all times. However, it can exceed the soil strength along the slice interface as it does not check the vertical interslice forces against the shear strength of the material. The results should therefore be checked for this criterion.

In the following sub-section, two case examples are discussed that were previously investigated to demonstrate the limitation of using Bishop method due to interlock.

### 3.2 Case examples

### 3.2.1 Example 1 (Figure 3.4, Chirapuntu & Duncan 1976)



Figure 4: Reproduced model of cohesive fill embankment on soft clay foundation (From Figure 3.4, Chirapuntu & Duncan 1976)

It should be noted that an imperial system of units is used for this example for easier comparison with the original study.

Slope analysis method	Factor of safety from	Factor of safety from Oasys
	Chirapuntu & Duncan (1976)	Slope (Oasys Ltd., 2022)
Ordinary	1.88	1.88
Bishop	1.55	1.55
Arup	-	1.88
Spencer	1.88	Solution did not converge
Morgenstern-Price	Solution did not converge	Solution did not converge

 Table 1: Summary of slope stability analysis for Example 1

Chirapuntu & Duncan (1976) exemplified the difficulty of employing the Bishop method investigating the results of slope stability analysis for an embankment made of highly cohesive soil (Figure 4). They performed slope stability analysis, using Ordinary, Bishop and Spencer methods, and noted good agreement between the factors of safety obtained for Ordinary and Spencer methods, while a much smaller factor of safety using Bishop method. The results from the original study are summarised in Table 1.

Referring to the formulation for effective normal force at the base of interslice above, we know that a high cohesion can result in a negative numerator, and thereby negative normal force at the base of some slices. This is not reasonable and would lead to the computation of an unreasonably low safety factor value.

This slope case was, hence, replicated and reran on Oasys Slope. The results were found to be consistent using both the Ordinary and Bishop methods, suggesting successful replication of the model. Interestingly, both Morgenstern-Price and Spencer methods resulted in convergence issues upon rerunning this example, unlike the original study which noted convergence issue for Morgenstern-Price, but not Spencer method. However, it should be noted that the authors also acknowledged difficulty in getting solution when using both Spencer and Morgenstern-Price methods, despite trying several interslice functions. The results are summarised in Table 1.

The model was then analysed using Arup method. A similar safety factor value was noted upon using the Arup method, when compared to the Ordinary and Spencer methods from original study.

Similar trends were also observed when the normal force on slip surface was plotted for the different slope methods. As stated in Chirapuntu & Duncan (1976), the unreasonably low factor of safety from Bishop can be attributed to the fact that the effective normal force on the bases of most of the slices in the fill are negative, shown in Figure 5.



Figure 5: Normal force on slip surface against slope distance





Figure 6: Reproduced model of stiff clay embankment on soft clay (From Figure 11, Low, 1989)

In this 'Illustrative Example 2' from Low (1989), an embankment of stiff clay was constructed on soft clay, and the undrained shear strength of the soft clay was noted to vary with depth (Figure 6). The

critical slip surface was determined using a computer program and the critical factors of safety were obtained using both Ordinary method of slices and the Bishop method.

Low (1989) noted a much lower factor of safety given by Bishop method, when compared against handcalculated safety factor value of 1.38 and the factor of safety computed using Ordinary method. Upon performing sensitivity check of friction angle using Bishop method, an unreasonable trend of increasing safety factor with reducing friction angle was also observed, which led to the conclusion that the safety factor from Bishop method is unreliable. The cause of the error was associated with an interplay between the steeply inclined bases of the slices as well as a high cohesion of the embankment material. In such circumstances, the normal force computed on the base of some slices can be suspected to either be negative or unreasonably large. The results from the original study are summarised in Table 2.

Slope analysis method	Factor of safety from Low (1989)	Factor of safety from Oasys Slope (Oasys Ltd., 2022)
Ordinary	1.31	1.33
Bishop	1.14	1.09
Arup	-	1.36
Spencer	-	Solution did not converge
Morgenstern-Price	-	1.41

 Table 2: Summary of slope stability analysis for Example 2

Hence, the model in this case example was replicated and reran on Oasys Slope. The model, upon rerunning, gave similar results for both the Ordinary and Bishop methods, indicating successful replication. Table 2 shows similar safety factor values of the slope model upon reanalysing using the two approaches. Additionally, the seeming paradox of the increasing safety factor upon reducing friction angle trend was also captured for the Bishop method and is shown in Figure 7.

The slope model was then analysed using the Arup, Spencer and Morgenstern-Price methods as well. Referring to the results shown in the table below, it can be seen that a factor value similar to the Ordinary Method was obtained when using Arup method, while a slightly higher safety factor was obtained when using Morgenstern-Price method. The Spencer method, however, could not converge. A sensitivity analysis was once again performed using Arup and Morgenstern-Price methods, and logical trends of safety factor against friction angle was noted for these methods, as shown in Figure 8.



Figure 7: Computed factor of safety under various friction angle values

Hence, it is evident that the Arup method, offered by Oasys Slope, solves slope stability problems involving interlock and gives reliable results. This new feature of enabling the inclination of the

interslice forces to vary is indeed a promising step forward towards performing accurate slope analyses and thereby delivering robust slope designs. Similarly, slope designs can benefit greatly if existing design procedures can be streamlined. Fortunately, Oasys Slope does offer designers the opportunity to automate using customised coding. In the following section, the authors will share their experience of developing optimisation tools using Oasys Slope and basic scripting to facilitate their design workflow involving soil nailing.

## 4 Automation In Soil Nailing Design

Soil nailing is an effective slope stabilization and upgrading method that is widely adopted in Hong Kong. With recent advancements in computation, optimization is possible through rapid parametric study and a streamlining of the analysis process. This section provides a detailed description of soil nail design automation using Oasys Slope and discusses how it leads to an efficient optimization of soil nail design.

## 4.1 Mapping design workflow

Mapping the existing design workflow can facilitate the identification of areas that can benefit most from design automation. The typical workflow for carrying out slope upgrading design is outlined in Figure 8. The workflow is separated into different stages. First step is the selection of critical section which is obtained based on the desk study and engineering judgement. The geometry and analysis method are then defined in a limit equilibrium program (such as Oasys Slope). Design parameters are entered manually through the GUI (graphical user interface), after which, Oasys Slope performs calculations to determine the critical slip surface and factor of safety for the slope.



Figure 8: Typical workflow for slope upgrading design

Using the results of the initial analysis, an engineer can determine whether the existing slope stability meets the required safety standards or requires upgrading or stabilization via soil nailing. It is important to note that the process of soil nail design involves several key parameters. For example, there can be multiple combinations for soil nail lengths, diameter, spacing and soil nail angle. In addition, the critical slip surface may change depending on the arrangement, length and other parameters. When soil nails are designed based on analytical approach, involving detailed calculation of bond strength based on specific slip surface, several manual calculations are required and have to updated whenever the soil nail parameters are modified or slip surface changes. Automating this crucial part of the design workflow can greatly assist an engineer and minimize chances of errors due to manual handling.

In Oasys Slope program, a single critical factor of safety is obtained per analysis setup and finding the optimal soil nail arrangement is an iterative process to test different patterns. The developed automation tool provides a way to do this iterative process automatically, through creating data sets in Excel and integrating it with Oasys Slope using scripts to control and run the analyses until an optimal design is found.

## 4.2 Reinforcement calculation in Oasys Slope

Oasys Slope allows the user to define several different types of reinforcement, including ground anchors, soil nails, geotextiles, and rock bolts. For active reinforcements specified, the forces in the reinforcement are calculated and applied in the analysis in slightly different ways depending on the selected analysis method. The soil nail function in Oasys Slope allows the specifications of the geometry of the soil nail systems, including the lengths, levels, inclinations, and out-of-plane spacing. The tensile capacity of the soil nails can also be specified. The bond strength of each soil nail is derived from the available bond length and the effective overburden stress of the current slip surface. The available capacity of each soil nail is then resolved into vertical and horizontal components, which are applied as surface loads in the calculations of the factor of safety.

Details on how reinforcement calculations are performed in Oasys Slope can be found in Oasys Ltd. (2022).

### 4.3 Automation with Oasys Slope

Oasys Slope 21.0 includes an API (Application Programming Interface) that allows the user to interact with the program using external VBA or Python scripts instead of going through the GUI (Graphical User Interface). Once the API has been setup, users can pass relevant data to Oasys Slope and remotely run analysis through calling its built-in COM functions. The full list of API COM functions can be found in the program manual Oasys Ltd. (2022).

This section presents a tool to automate the iterative design process using Visual Basic (VBA) programming language. VBA is complimentary and built into Microsoft Office, thus Excel is chosen as an intermediary to store data and act as a control interface for Oasys Slope through VBA scripts.

Key advantage in using Excel is that design information can be inputted and stored in an accessible and familiar format, RANGE functions within VBA scripts can be used to specify the data input area within each worksheet and feed the data to Oasys Slope through COM functions in the VBA scripts. The same COM functions can be used to extract the output from Oasys Slope once each analysis is completed and pass the data back to Excel, where additional processing, such as applying factors or comparing between the last results, can be done from within Excel easily.

The Oasys Slope analyses are setup to loop within the VBA scripts until the termination criterion is reached.

This can be set within excel using a "switch cell", which is an IF statements that compares the analysis result and ends the loop if the conditions are met, for example, if the minimum factor of safety falls below 1.

An optimization function is made for soil nails using VBA, which sequentially modifies each soil nail length by a user-specified interval until the required factor of safety is achieved. Using more rigorous optimization algorithms, by adding additional boundary conditions checks, for e.g., applying soil nail length constraints, or using more advanced search method altogether, such as the gradient descent method, can produce greater accuracy in soil nail design and allow more flexibility in soil nail optimization by allowing alternating nail lengths. Other boundary conditions, such as water table height or surcharge can similarly be manipulated and ran successively by implementing their respective COM functions in the VBA scripts and modifying the termination criteria switch cell within Excel to include their consideration. This enables rapid parametric study to better understand the influence of each factor on the nail design separately.

Outputs can also be extracted by calling the COM functions, which will save the data to sets of variables (e.g. FOS1, FOS2 etc.) to be printed in the user desired format in Excel. The results can then be used for further analysis or presentation of results. For checking of model analysis, COM functions can command Oasys Slope to save a new .sbd file after each iteration in the loop as record. An outline of the automation framework is shown in Figure 9.



Figure 9: Oasys Slope automation framework

## 4.4 Benefits of automation

By enabling data to be passed automatically to and from Oasys Slope, efficiency in carrying out analyses is substantially increased. This allows a large number of design scenarios to be analysed through iterative looping and setting different termination criteria.

Increased model output allows a more comprehensive understanding of the slope's condition to be visualized, an example is demonstrated below with the relationship between soil nail length and critical factor of safety. This also serves as a sensitivity check and similar reference data can be extracted from each iteration of the model to lend more confidence to the engineer's design.

Designers can create a data set of parameters from envisioning the possible design conditions and quickly analyse all of them through the automation tool. The tool can automatically find solutions that satisfies multiple design constraints with high precision, in addition, the automation also removes the need to manually input data into each model through the GUI, reducing the chance of human error, resulting in overall higher accuracy.

Optimization algorithms can be implemented within the automation framework, allowing the engineer to refine their designs automatically. However, it should be noted for soil nail design, various nail patterns can give the same factor of safety. Having a good understanding in the selected optimization algorithm as well as exercising engineering judgement to determine a reasonable nail pattern will be required to produce more feasible design.

Nevertheless, the automation is still capable in producing, if not exact, then a very close first estimate of slope upgrading works required, streamlining the soil nail design process. The framework can also quickly produce updated results in the event of design parameter changes, promoting flexibility.

Ultimately, using the automation for slope design is expected to produce optimized and high-quality designs in less time.

## 4.5 Demonstration of Soil Nail optimization using automation

An example is shown in Figure 10 below to demonstrate the automation's capability to optimize soil nail design and produce large number of models for sensitivity checking.

The given feature is part of a project to assess the effects of varying soil nail lengths on slope factor of safety. The feature's geometry and design parameters were reviewed from desk study and setup in Oasys Slope.

The procedure to carrying out the soil nail optimization was as follows:

- 1. The material and surcharges were tabulated in Excel and passed to Oasys Slope through COM functions.
- 2. Two base versions of the Slope model, one with a design water table and the other with a worst ground water table was created to check for both cases.
- 3. Specified slip surfaces were setup to distinguish the critical slip more clearly and to reduce the run time, though it is possible to use grid and radius method with the automation.
- 4. The soil nails were optimized through reducing each nail's length incrementally by 1m using WHILE loops in VBA until required factor of safety is reached.
- 5. The Factor of safety for each of the soil nail patterns tested in the analysis loop are extracted to Excel and plotted to generate a full relation between soil nail length and the critical factor of safety for each of the water table cases, shown in Figure 11 and 12.



Figure 10: Feature with soil nails setup in Oasys Slope



Figure 11: Soil nail length against factor of safety (1-in-10 design ground water table condition)



Figure 12: Soil nail length against factor of safety (predicted worst ground water table condition)

A total of 48 combinations of soil nails were analysed with the automation. A complete behaviour can be obtained from plotting the data, it can be observed that for the 1-in-10 design water table case, the factor of safety increases linearly with increasing soil nail length. For the predicted worst ground water table case, there is a gap around FOS = 1, attributed to the higher water table and its proximity to those slip surfaces, the automation thus allows us to know if there are sensitive ranges to be avoided during design.

## 5 Conclusion

The use of variably inclined interslice forces in the slope stability analysis (i.e. Arup method) based on limit equilibrium method is presented. Its computer implementation solves slope stability problems involving interlock and gives reasonable results. It is, however, imperative to check the details of a slope stability analysis results to ensure that the solution does not violate the assumptions made in the analysis procedure used. A simple reliance on the computed safety factor alone to a slope problem without scrutinizing other details of a solution should be avoided under all circumstances.

This paper also presented our development of a digitalised workflow for slope analyses that allows streamlining of the analysis process and optimisation through rapid parametric study. The use of Oasys

Slope software along with customised coding enables automation of the design workflow and optimisation of soil nailing design.

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#### References

- Bishop, A.W. 1955. The use of the slip circle in the stability analysis of earth slopes. Géotechnique, 5(1): 7-17. https://doi.org/10.1680/geot.1955.5.1.7
- Chirapuntu, Suphon., & Duncan, J. M. James M. (1976). The role of fill strength in the stability of embankments on soft clay foundations. In *This Digital Resource was created from scans of the Print Resource*. Soils and Pavements Laboratory (U.S.). https://erdc-library.erdc.dren.mil/jspui/handle/11681/20519
- GEO. (2007). Review Of Limit Equilibrium Methods for Soil Nail Design. https://www.cedd.gov.hk/filemanager/eng/content\_389/er208links.pdf
- Janbu, N. (1957). Earth Pressures and Bearing Capacity Calculations by Generalized Procedure of Slices. *Proceedings of the 4th International Conference on Soil Mechanics and Foundation Engineering*, 207–212. https://www.issmge.org/publications/online-library
- Low, B. K. (1989). Stability Analysis of Embankments on Soft Ground. Journal of Geotechnical Engineering, 115(2), 211–227. https://doi.org/10.1061/(ASCE)0733-9410(1989)115:2(211)
- Morgenstern, N. R., & Price, V. E. (1965). The Analysis of the Stability of General Slip Surfaces. *Geotechnique*, 15(1), 79–93. https://doi.org/10.1680/GEOT.1965.15.1.79
- Oasys Ltd. 2022. Slope 21.0 Manual. https://www.oasys-software.com/public/ProductHelp/slope/21.0/Slope21.0\_Manual.pdf
- Spencer, E. 1967. A method of analysis of embankments ensuring parallel interslice forces. Géotechnique, 77: 11-26. https://doi.org/10.1680/geot.1967.17.1.11
- Whitman, R. V., & Bailey, W. A. (1967). Use of Computers for Slope Stability Analysis. Journal of the Soil Mechanics and Foundations Division, 93(4), 475–498. https://doi.org/10.1061/JSFEAQ.0001003
- Wright, S. G., Kulhawy, F. H., & Duncan, J. M. (1973). Accuracy of Equilibrium Slope Stability Analysis. Journal of the Soil Mechanics and Foundations Division, 99(10), 783–791. https://doi.org/10.1061/JSFEAQ.0001933