

# A Note on Subgroups of the Symmetric Group $S_5$ and Some Lower Intervals of $L(S_5)$

R. Sathiyapriya and Sathesh Kumar N.\*

Department of Mathematics, Dhanalakshmi Srinivasan University, Samayapuram, Trichy, India

\*Corresponding author: satheshmhss@gmail.com, sathiyap.set@dsuniversity.ac.in

## ABSTRACT

The symmetric group  $S_5$  is the group of all permutations on the set containing 5 elements. Since the symmetric group of order  $n$  has  $n!$  elements, so there are 120 elements in the symmetric group  $S_5$ .

We can list out all the subgroups of  $S_5$  according to their orders. As per Lagrange's theorem, the order of any non-trivial subgroup of  $S_5$  divides the order of  $S_5$ . Obviously, the only subgroup of  $S_5$  of order 1 is the trivial group. Since there exist subgroups of orders. Consequently, there is an order 2 subgroup. Since 2 is a prime number, the subgroup is cyclic, and it is generated by the elements of order 2. Thus, there are 25 subgroups of  $S_5$  of order 2. Since, and  $\dagger$ , has a 3 – sylow subgroup of order 3. The number of 3 – Sylow subgroups are of the form, that is, The possible for  $k$  are 0, 1, and 3. Therefore the maximum number of 3 – Sylow subgroup of order 3 is 10 when  $k = 3$ .

The subgroup of  $S_5$  of order 4 contains the elements of order 1, 2, or 4. There are 15 subgroups of  $S_5$  of order 4 containing elements of order 4. If the subgroup contains an element of order 2, then it does not contain an element of order 4. Therefore, there are 20 subgroups of order 4 containing elements of order 2 only.

**Keywords:** Symmetric group, subgroups, Lattice theoretic properties

