On the μF-subgroups of Some Finite Abelian Groups

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ABSTRACT

The paper presented here introduces and explores the concept of the subgroup determined by Möbius function, denoted as the μF-subgroup, within the context of finite cyclic groups C n. It makes significant contributions to the field of group theory by investigating the properties and relationships of these μF-subgroups within different group structures. One of the primary findings of this paper is the assertion that within finite cyclic groups C n , the collection of all μF-subgroups, denoted as LμF(C n), forms a sub lattice of the lattice L(C n). This result is notable because it establishes a specific structure within the lattice of subgroups of cyclic groups. Furthermore, the paper identifies a fundamental connection between Hall subgroups and μF-subgroups, emphasizing that every Hall subgroup of a group qualifies as a μF-subgroup. This connection sheds light on the broader relevance and significance of μF-subgroups in group theory. The paper extends its investigation to the product of cyclic groups, C m × C n, and explores the meet and joins operations of subgroups within this product group. It proves that the lattice of μF-subgroups, denoted as LμF(C m × C n), is not necessarily a sub lattice of the lattice L (C m × C n). However, the paper provides the condition when LμF(C m × C n) forms a lattice, and the methods to determine the meet and join of any two μF-subgroups within this context. A significant contribution of the paper lies in establishing a characterization for LμF(C m ×C n) to be a sub lattice of L(C m × C n) and specifying the conditions under which this occurs. This characterization adds depth to our understanding of when and how sub lattices can be formed within the lattice of subgroups in a product group. Finally, the paper explores the cardinality of the set LμF(C m × C n) for various values of m and n, providing insights into the size and complexity of these μF-subgroups within the product group.

Keywords: Lattice, Groups, μF-subgroup

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