

Enhanced Semi-local Convergence Analysis for Higher-order Iterative Techniques

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ABSTRACT

Iterative methods play a pivotal role in solving nonlinear equations, optimization problems, and numerical simulations across various scientific and engineering domains. This paper investigates the semi-local convergence behavior of a fourth order iterative technique, aiming to enhance its applicability. In this paper, we embark on an exploration of semi-local convergence analysis, delving into its crucial role in comprehending the convergence dynamics of higher order iterative algorithms. Departing from traditional methodologies necessitating Taylor series expansions replete with higher order derivatives, our approach exhibits ingenuity and novelty. Our newly devised convergence theorems exclusively rely on first order derivatives. This innovation significantly expands the operable scope of the method, curtailing limitations on its working range, and concurrently curbing the computational expenses associated with solution determination. We broach the concept of semi-local convergence, an indispensable facet that enhances our comprehension of algorithmic behavior. In comparison to broader convergence considerations, semi-local convergence provides a more precise demarcation of the regions in which iterates naturally inhabit. This pinpoint accuracy helps practitioners with a deeper understanding of the algorithm's behavior, augmenting its suitability for various applications. Our approach shuns the conventional reliance on high order derivatives, introducing an innovative framework that not only preserves convergence order but also enriches our understanding of the convergence domain's characteristics and error dynamics. The presented analysis sheds light on the practical behavior of these methods and paves the way for designing more efficient and robust algorithms for a wide array of computational tasks.

Keywords: Non-linear system, Semi-Local Convergence, Banach Space

