Phonon Modes χ3 Non-linearity in Ion Chains

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Trapped ions are one of the most promising systems suitable for quantum computation. In Coulomb crystals consisting of trapped ions, high-fidelity single-qubit and entangling gates have been demonstrated, and quantum simulations of long-range Ising spin chains have been performed. To generate couplings and create entanglement between different ions, collective vibrational excitations (phonon modes) of the Coulomb crystal are utilized. Due to intrinsic nonlinearity of the Coulomb interaction, there are various types of nonlinear interactions between the phonon modes. They are one of the sources of the gate's infidelity in the ion chain, and they can also be used to perform quantum nondemolition measurements.

In this work, we study the Kerr-type nonlinearities of the phonon modes of the linear Coulomb crystal. The interaction part of the phonon modes Hamiltonian with account for the Kerr and cross-Kerr terms reads $H = \sum_{\alpha,\beta,\gamma} c^{(1)}_{\alpha,\beta,\gamma} \grave{a}^{\dagger}_{\alpha}\grave{a}^{\dagger}_{\beta}\grave{a}^{\dagger}_{\gamma} + \sum_{\alpha,\beta,\gamma} \hbar\omega^{(2)}_{\alpha,\beta,\gamma} \grave{a}^{\dagger}_{\alpha}\grave{a}^{\dagger}_{\beta}\grave{a}_{\gamma} + \sum_{\alpha,\beta,\gamma} c^{(3)}_{\alpha,\beta,\gamma} \grave{a}^{\dagger}_{\alpha}\grave{a}_{\beta}\grave{a}_{\gamma} + \sum_{\alpha,\beta,\gamma} c^{(4)}_{\alpha,\beta,\gamma} \grave{a}_{\alpha}\grave{a}_{\beta}\grave{a}_{\gamma} + \sum_{\alpha,\beta} c^{(5)}_{\alpha,\beta} \grave{a}^{\dagger}_{\alpha}\grave{a}_{\alpha}\grave{a}^{\dagger}_{\beta}\grave{a}_{\beta}$, where: $\grave{a}^{\dagger}_{\alpha}, \grave{a}_{\alpha}$ are the creation and annihilation operators of the normal mode α, and $c^{(i)}_{\alpha,\beta,\gamma}$- are constants. To find these coefficients for the Coulomb crystal consisting of N ions, we considered the crystal Hamiltonian

$$H = \sum_{j=1}^{j=N} \left(\frac{p^2_{x,j} + p^2_{y,j} + p^2_{z,j}}{2m} + \frac{m\left(\omega^2_{rad} x^2_j + \omega^2_{rad} y^2_j + \omega^2_{ax} z^2_j\right)}{2} \right) + \frac{e^2}{8\pi\epsilon_0} \sum_{j,m=1}^{j,m=N} \frac{1}{\sqrt{\left(x_j - x_m\right)^2 + \left(y_j - y_m\right)^2 + \left(z_j - z_m\right)^2}}$$

where x_j, y_j, z_z are the positios of the ion j, $p_{x,j}, p_{y,j}, p_{z,j}$ are the momenta of the ion j along the axes X, Y, Z, m is the mass of the ion; $\omega_{rad}, \omega_{ax}$ are the radial and axial frequencies of the confining potential. We developed a numerical algorithm to calculate the ions equilibrium positions and found the normal modes spectrum. Then, we expanded the Coulomb potential in Taylor series up to the fourth order in the deviations from the equilibrium positions and expressed the expansion terms through the phonon creation/annihilation operators. For the fourth order, the Kerr and cross-Kerr terms which not change the number of quanta cause the most considerable effect.

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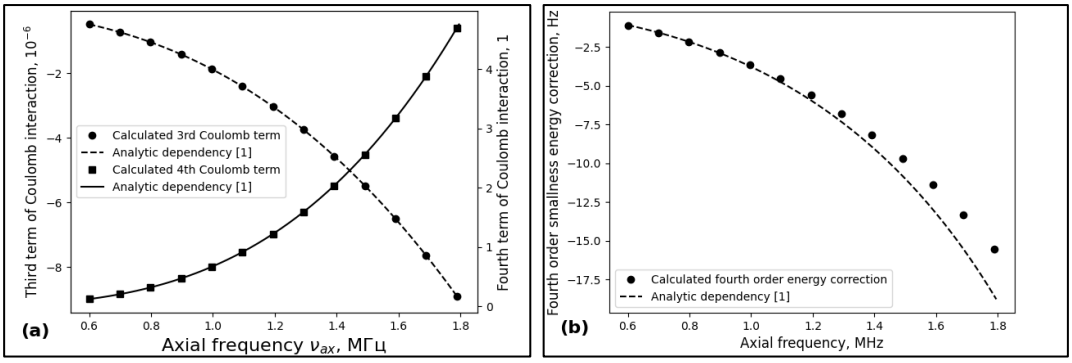

Fig.1: (a) *Coulomb interaction coefficients for axial modes calculated and analytic solution for 2 ion chain and.* **(b)** *Frequency shift correspond fourth order energy correction. Assumed radial frequency is 4 MHz and* $^{40}_{20}Ca^{+}$ *ion.*

Using the obtained coefficients of the nonlinear interactions, we found the corrections to the energies of the phonons Fock states. Also, we compared the third and the fourth orders of the numerically obtained Hamiltonian expansion Coulomb terms the with analytic solution for the two-ion chain [1] presented in Fig.1. For the energy corrections of the fourth order, the comparison with the analytic solution for the two-ion chain [1] are presented in Fig.2. In both cases, there is satisfactory coincidence between the analytical and numerical results. Also, we found the scaling law of the Kerr coupling in the Coulomb crystal.

References

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