Embedding of a Signed Graph with Property *P* in a Graceful Signed Graph with Property *P*

Jessica Pereira[1*], Tarkeshwar Singh[2], S. Arumugam[3]

[1]School of Physical and Applied Sciences, Goa University, Goa-403206, India
[2]Department of Mathematics, Birla Institute of Technology and Science, Pilani, Goa Campus
[3]National Centre for Advanced Research in Discrete Mathematics, Kalasalingam University, Anand Nagar, Krishnankoil-626 126, Tamil Nadu, India

*Corresponding Author

ABSTRACT

Let $S = (V, E, s)$ be a signed graph with $|V|=p$, $|E|=q$ and let $s: E \rightarrow \{+, -\}$ be a function which assigns a sign + or - to each edge. For any injection $f: V \rightarrow \{0, 1, ..., q\}$, the induced edge labelling g_f is defined by $g_f(uv)=s(uv)|f(u)-f(v)|$. The function f is said to be a graceful labelling of S, if $g_f(E^+) = \{1, 2, ..., |E^+|\}$ and $g_f(E^-) = \{-1, -2, ..., -|E^-|\}$ where E^+ and E^- denote the set of all positive and negative edges of S respectively. A signed graph that admits graceful labelling is called a graceful signed graph. In this paper, we prove that a signed graph S having property P can be embedded in a graceful signed graph S' having property P when P denotes the property being: triangle-free, planar, Eulerian, or Hamiltonian. We have also proved that if S is a connected graph and S_1, S_2, ... S_k is its decomposition into edge-induced subgraphs with $f: V \rightarrow N \cup \{0\}$ an injection having maximum vertex label $M_{S(f)}$, such that the edge-induced function g_f assigns distinct labels to edges of S_i, $1 \leq i \leq k$. Then S can be embedded as an induced subgraph of k-hypergraceful eulerian graph S' with k-hypergraceful labelling h such that $M_{S'}(h) \leq 2^{k+1}(M_{S(f)}+4)-7$.

Keywords: Signed graph, Graceful signed graph, Graceful embedding.

.