

Modeling and Analysis of Porous Medium Equations: Existence-uniqueness Results and Asymptotics

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ABSTRACT

We introduce basic governing equations for fluid flow inside porous medium along with specific applications. We focus our attention to Brinkman-Forchheimer equation that involves a non-linear term. We first define a boundary value problem of flow inside a bounded domain and show the corresponding weak formulation. We then convert the same into equivalent fixed-point problem and establish the existence and uniqueness results via Banach contraction theorem. We then consider a fully developed flow corresponding to the anisotropic Brinkman-Forchheimer extended Darcy's equation in a channel packed with porous medium. We discuss existence and uniqueness results using the Browder-Minty theorem. Complemented with the existence and uniqueness analysis, we present an asymptotic solution by taking Darcy number as the perturbed parameter. For a high Darcy number, the corresponding problem is a regular perturbation expansion. For low Darcy number, the problem of interest is a singular perturbation. We use matched asymptotic expansion to treat this case. More generally, we obtained an approximate solution for the non-linear problem, which is uniformly valid irrespective of the porous medium parameter values. The analysis presented serves a dual purpose by providing the existence and uniqueness of the anisotropic non-linear Brinkman-Forchheimer extended Darcy's equation and provides a more robust approximate solution. We discuss specific results within the context of flow inside endothelial glycocalyx layer.

Keywords: Banach contraction, endothelial glycocalyx layer, Browder-Minty theorem.

