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Real Time Ultrasoft Longitudinal Photons Self Energy at Next to-leading Order in Hot Scalar QED

Amel Youcefi¹, Karima Bouakaz^{1*}, Abdessamad Abada²

¹Department of Physics, Laboratoire de Physique des Particules et de Physique Statistique,
Ecole Normale Supérieure, BP 92 Vieux-Kouba, Algiers

²Department of Physics, United Arab Emirates University, P. O. B. 17551, Al Ain, United Arab Emirates

*Corresponding Author

ABSTRACT

We determine a compact analytic expression for the complete next-to-leading contribution to the retarded Longitudinal Photons self-energy with ultrasoft momentum in the framework of hard-thermal-loop (HTL) -summed perturbation of massless Scalar QED at high temperature. The calculation is done using real-time formalism. The real part and the opposite of the imaginary part of the retarded Longitudinal Photons self-energy are related to the next-to-leading order contributions of energy and damping rate respectively.

Keywords: Soft Photons Energy and Damping Rate; Resummation; Hard Thermal Loop

Introduction

In recent years, there has been steadily increasing activity aimed at analyzing the phase structure of Quantum Chromodynamics (QCD), the quantum field theory of strong interactions. A detailed understanding of the properties of the deconfined phase of QCD, the so-called Quark-Gluon Plasma, is important in several areas of Physics. Examples are the evolution of the early Universe [1], heavy ion collisions at RHIC (Brookhaven) and LHC (CERN) [2]. A major problem that spurred progress in the late 1980's was the apparent gauge dependence of the one-loop order quasiparticle dispersion relations, most notably the gluon damping rate. The problem was resolved in [3]. It turns out that in order to calculate consistently at high temperature, we have to use an effective perturbation that sums the so-called hard thermal loops (HTL) into dressed propagators and vertices [3, 4]. The first next-to-leading order physical quantity that has been determined in the framework of the HTL program is the zero-momentum transverse gluon damping rate [5]. It was shown to be finite and positive. Subsequent studies of the behavior of the gluon and quark damping rates in the imaginary-time formalism have indicated that there are difficulties in the infrared sector [6-12]. A similar observation has been done in the context of scalar electrodynamics [13]. To look further into the infrared behavior, we propose to calculate the next-to-leading order dispersion relations for slow-moving Longitudinal Photons at high-temperature scalar quantum electrodynamics (Scalar QED), using the real time formalism (RTF) in physical representation. We derive the analytic expressions of hard thermal loop (HTL) contributions to propagators to determine the expressions of the effective propagators in RTF that contribute to the complete next-to leading order contribution of retarded Longitudinal photons self-energy. The



longitudinal retarded photons self-energy is related to the next-to-leading order dispersion relations.

Effective expansion

In the Landau gauge, the effective photon propagator followed from the resummation of the HTL photon self energy. To leading order the effective photon propagator is given by:

$$\Delta_{ra/ar}^{\mu\nu}(K) = P_T^{\mu\nu} \frac{1}{\delta\Pi_T^{r/a} - K^2 \mp i\text{sgn}(k_0)\varepsilon} + P_L^{\mu\nu} \frac{1}{\delta\Pi_L^{r/a} - K^2 \mp i\text{sgn}(k_0)\varepsilon}, \quad (1)$$

where $P_T^{\mu\nu}, P_L^{\mu\nu}$ are the usual transverse and longitudinal projectors respectively and

$$\begin{aligned} \delta\Pi_L^{r/a}(K) &= -3m^2 \left[1 - \frac{k_0}{2k} \ln \frac{k_0 + k \pm i\varepsilon}{k_0 - k \pm i\varepsilon} \right] \\ \delta\Pi_T^{r/a}(K) &= \frac{3}{2} m^2 \frac{k_0^2}{k^2} \left[1 - \left(1 - \frac{k^2}{k_0^2} \right) \frac{k_0}{k} \ln \frac{k_0 + k \pm i\varepsilon}{k_0 - k \pm i\varepsilon} \right] \end{aligned} \quad (2)$$

with $m = \frac{1}{6} e^2 T^2$ the photon thermal mass.

The HTL contribution to the scalar self-energy is obtained by summing the one loop diagrams and it is given by:

$$\delta\Sigma_{htl}(K) = m_s^2, \quad (3)$$

with $m_s = eT/2$ the scalar thermal mass.

The HTL resummed scalar propagator is given by

$$\begin{aligned} \Delta_{R,A}^{-1}(K) &= K^2 - m_s^2 \pm i\text{sgn}(k_0)\varepsilon \\ \Delta_F^{-1}(K) &= -2\pi i [1 + 2n_B(|k_0|)] \delta((K^2 - m_s^2)), \end{aligned} \quad (4)$$

where $K = (k_0, k)$ and $n_B(k_0) = 1/(\exp(k_0/T) - 1)$ the Bose distribution function.

The dressed vertices are equal to the tree amplitudes, i.e., unaffected by the hard thermal loops.

The vertex with one photon and two scalar external lines undressed (Q incoming, P outgoing) is:

$$\Gamma^\mu(P, Q) = -e(P + Q)^\mu, \quad (5)$$

and the vertex between two photons and two scalars is momentum independent and writes:

$$\Gamma^{\mu\nu}(P, Q) = -ie^2 g^{\mu\nu}, \quad (6)$$

Damping rate and energy for photons in hot SQED

The dispersion relations are defined by:

$$p^2 - \delta\Pi_{ra/ar}^l(\Omega_l, p) - {}^*\Pi_{ra/ar}^l(\Omega_l, p) = 0, \quad (7)$$

where ${}^*\Pi^l$ is the next-to-leading order longitudinal photon self-energy. The next-to-leading-order energy of the slow-moving longitudinal photons is:

$$\text{Re}\Omega_l(p) = \omega_l(p) - \frac{\text{Re} {}^*\Pi_{ra/ra}^l(P)}{\left. \frac{\partial}{\partial\omega} \delta\Pi_{ra/ar}^l(P) \right|_{\omega=\omega_l}} \quad (8)$$

and the damping rate is given by:

$$\gamma_l(p) = - \frac{\text{Im} {}^*\Pi_{ra/ra}^l(P)}{\left. \frac{\partial}{\partial\omega} \delta\Pi_{ra/ar}^l(P) \right|_{\omega=\omega_l}} \quad (9)$$

where ω_{s_0} is the leading-order scalar energy. So, to obtain the next-to leading-order dispersion relations for slow moving longitudinal photons, we have to determine the next-to leading-order 'NLO' longitudinal photons self-energy. The diagrams that contribute to next-to-leading-order longitudinal photons self energy are the following two diagrams, in which the internal momenta are soft.

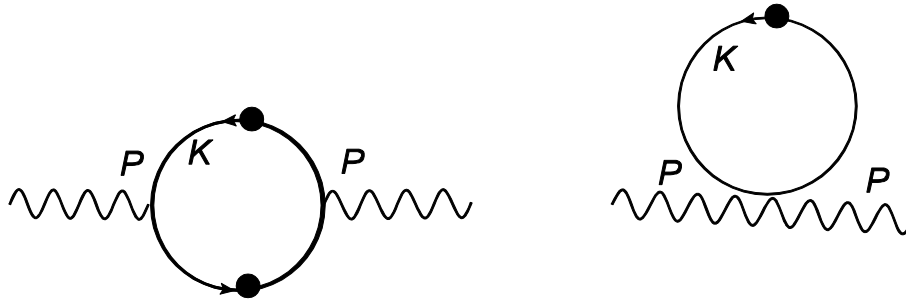


Figure1: NLO HTL-summed longitudinal photons self-energy

The contribution of the two diagrams in the Keldysh basis is given by:

$$\begin{aligned} {}^*\Pi_{ab}^l(P) = & \int \frac{d^4 K}{(2\pi)^4} \left[{}^*\Gamma_{ab\alpha\beta}^{\mu\nu}(Q, -Q, K, -K) {}^*\Delta_{\alpha\beta}(K) \right. \\ & \left. + {}^*\Gamma_{\alpha\alpha\beta}^{\mu}(K, K-Q) {}^*\Delta_{\alpha\alpha'}(K) {}^*\Gamma_{\alpha'\beta\beta'}^{\nu}(K-Q, K) {}^*\Delta_{\beta\beta'}(K-Q) \right] \end{aligned} \quad (10)$$

where K is the soft internal momentum and $Q=P-K$.

In Keldysh representation the retarded, advanced and symmetric resummed (effectif) propagator are given by:

$$\begin{aligned} {}^*\Delta_{r,a}(K) &= \frac{1}{K^2 - m_s^2 \mp i \text{sgn}(k_0) \epsilon}, \\ {}^*\Delta_s(K) &= -2\pi \left(1 + 2n_B(|k_0|) \right) \delta(K^2 - m_s^2) \end{aligned} \quad (11)$$

where we have used the notation $K = (k_0, \vec{k})$, $k = |\vec{k}|$ and $n_B(x) = 1/(\exp(x/T) - 1)$ denotes the Bose

distribution function. The effective temperature dependent scalar mass is given by $m_s = \frac{eT}{3}$. The contribution of the first term using the real time formalism to the retarded self energy can be written as:

$${}^{tad*} \Pi_R^L(P) = -ie^2 \int \frac{d^4 K}{(2\pi)^4} \left({}^* \Delta_F(K) + {}^* \Delta_R(K) + {}^* \Delta_A(K) \right) \quad (12)$$

After integrating over k_0 by means of δ function and trivially over the angle reduces to:

$${}^{tad*} \Pi_R^L(P) = -\frac{e^2}{\pi^2} \int_0^\infty dk \frac{k^2}{\omega_k} n_B(\omega_k) \quad (13)$$

Where $\omega_k^2 = k^2 + m_s^2$

The contribution of the second term is given by:

$${}^{1*} \Pi_R^L(P) = \frac{e^2}{2} \int \frac{d^4 K}{(2\pi)^4} (2k_0 + p_0)^2 \left[{}^* \Delta_S(Q) {}^* \Delta_R(K) + {}^* \Delta_A(Q) {}^* \Delta_S(K) \right] \quad (14)$$

Conclusion

In this work, we have reported on the progress in our determination of the HTL next-to-leading Longitudinal Photons dispersion relation. We have derived a compact analytic expression for the complete next-to-leading contribution to the retarded longitudinal photons self-energy in the context of hard-thermal-loop summed perturbation of Scalar QED at high temperature using real time formalism. These expressions need to be manipulated, mainly numerically, to determine the next-to-leading order longitudinal photons dispersion relation. This work is in progress.

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