

# Comparison of Analytical Models for Seismic Induced Pounding in Buildings – A Review

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doi: <https://doi.org/10.21467/proceedings.112.38>

## ABSTRACT

During an earthquake, buildings will have a tendency to sway too much and when the relative displacements between the buildings become greater than their separation gap, collision occurs. This results in serious damage or sometimes destruction of the structure. This phenomenon is termed as seismic pounding. In cities, because of large number of occupants and high land values, buildings are being constructed without enough separation gap between two buildings. Many numerical models were proposed over the past few years for calculating the force of collision during pounding. In this paper, a comparative study based on efficacy of various contact models like linear spring, linear viscoelastic (Kelvin-Voigt), modified linear viscoelastic, Hertz non linear, Hertz damp and non linear viscoelastic models are carried out. In addition to this, a study on a model proposed by Wang et.al. (Wang model) to analyse low speed pounding between viscoelastic material and steel is also carried out. The results of the study indicated that, Hertz damp model is considered to be the best model for simulating the pounding behaviour generally. But in case of low speed pounding (at range from 0.025 to 0.15m/s) Wang model is best suited.

**Keywords:** collision, pounding, contact models, low speed pounding, Wang model.

## 1 Introduction

Earthquake is the most disastrous of all natural catastrophes due to its huge power of devastation and total unpredictability. This causes collision of adjacent structures which may lead to mild damages or even collapse. The severity of the collision depends upon characteristics of earthquake like magnitude, acceleration, etc. Numerous research works have been conducted worldwide in the last few decades to investigate the causes of failure of different types of buildings under severe seismic excitations. Earthquakes impart lateral loads on the structure because of which the structure tend to dynamically sway. In the case of buildings built near to one another, without necessary seismic gap, they will have a tendency to collide with each other because of their difference in dynamic characteristics [1]. This phenomenon is known as pounding. This is quite common in urban areas where buildings are being constructed without providing required gaps. Pounding results in the generation of additional stresses, shear forces and collision forces in the buildings. But it is not compulsory to provide separation distance for buildings having identical period [4]. The closely spaced structures in our modern cities create a favourable condition for seismic pounding. The collision force decreases with increase in building height [5]. It is evident that seismic pounding is a highly dangerous phenomenon that has to be avoided or mitigated.

The aim of this study is to analyse and compare different numerical models which are proposed to simulate earthquake-induced structural pounding. Some of the commonly used numerical models are linear-spring



model, Kelvin-Voigt model, modified Kelvin-Voigt model, Hertz non-linear elastic model, Hertz damp model, non-linear viscoelastic model and Wang model (generally used for low speed pounding).

## 2 Modelling of earthquake induced structural pounding

Structural pounding is a complex phenomenon. It involves local crushing, plastic deformation, and damage at contact regions. It is very difficult to model these non linear deformations occurred during pounding [2]. Accurate modelling of earthquake induced pounding is inevitable to study the effect of collision between two structures. The two different methods used for modeling of pounding are classical theory of impact (stereomechanical approach) and force based approach [13]. Classical theory of impact is based on the laws of conservation of energy and momentum. The occurrence of stresses and deformations in colliding structural systems during the event of collision is not considered in this approach. This approach is not taken as a force based approach, the collision effect is reported by taking the velocities of the structural elements considered. Since stereomechanical approach neglects the period of impact, it is too rarely adopted in problems related to structural pounding, presuming that it stays only for negligibly small duration of time, therefore this approach does not calculate the contact force during collision.

The direct model of contact force during the collision is used to simulate the earthquake-induced structural pounding in force based approach. A spring with stiffness is used to simulate the impact stiffness of the colliding buildings is first introduced in this approach [13]. Once the adjacent building structures come in contact with each other, the spring gets activated.

### 2.1 Equation of motion for pounding

During the process of pounding between two structures, unlike the response for an independently vibrating individual structure, besides damping ratio, it depends on the masses and gap in between the structures also. Equation of motion for the MDOF systems during pounding due to seismic excitation can be written as [3].

$$M\ddot{u} + C\dot{u} + Ku + F_C(t) = -M\ddot{u}_g$$

Where  $M$ ,  $C$ ,  $K$  is  $n \times n$  mass, damping and stiffness matrices respectively.  $\ddot{u}_g$  is the ground movement during acceleration.  $F_C(t)$  is a vector depicting the pounding forces at the floor level. Appropriate numerical model of pounding forces  $F_C(t)$  should be used during the collision between buildings is vital for the accurate calculation of the pounding force response.  $\dot{u}$ ,  $\ddot{u}$  and  $u$  are the velocity, acceleration and displacement relative to the ground respectively. The dot denotes differentiation with respect to time,  $t$ .

### 2.2 Different phases in modelling of pounding

There are two phases in pounding force time history during impact. The first phase is known as the approach period. In this period, the curve gradually rises when the gap between two neighboring buildings becomes zero. It increases until the maximum deformation is attained. At the instant of detachment, the restitution period begins. The colliding elements are within the elastic range at the primary stage of approach period, but, subsequently, local cracking or crushing, plastic deformations usually occurs. During restitution phase, the gathered elastic strain energy is freed without extensive plastic effects in that phase. During impact, the majority of the energy is dissipated is lost during the approach period of collision. Very small amount of energy is energy is dissipated during the restitution period. Based on the experimental results, the force vs. time graph shows a rapid increase in pounding force during the approach period while, during the restitution period, the pounding force decreases in a lower rate as shown in Figure 1.

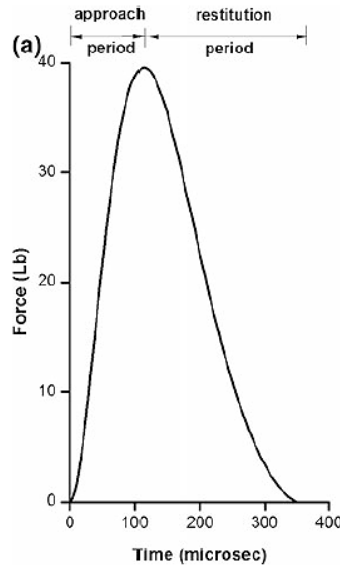


Figure 1: Force vs. time relation. [12]

### 2.2.1 Linear spring model

In this model, the impact element comprises of a linear elastic spring (Figure 2 (a))[8]. A linear spring of stiffness ( $k$ ) is used to simulate impact once the distance between neighboring structures decreases. The impact force–displacement relation is as shown in Figure 2 (b) and it can be represented as follows [8].

$$F_c = k(u_1 - u_2 - g_p); u_1 - u_2 - g_p > 0$$

$$F_c = 0; u_1 - u_2 - g_p \leq 0$$

Where  $F_c$  is the contact force and  $(u_1 - u_2 - g_p)$  is the relative penetration. This method can be easily utilized in commercial software. Disadvantage of linear elastic model is that it will not consider energy dissipation during collision.

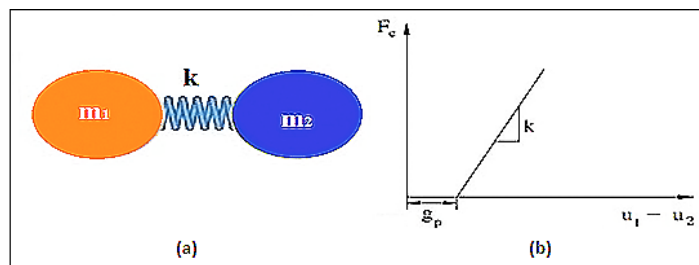


Figure 2: (a) linear spring model [15] (b) Force-displacement graph of linear spring model [8]

### 2.2.2 Linear viscoelastic model (Kelvin-Voigt model)

The drawback of linear spring model is resolved in the linear viscoelastic model (Kelvin-Voigt model) [8]. This model considers energy dissipation during collision. In this model the impact element comprises of a linear spring of stiffness  $k$  in addition to a linear damper as shown in Figure 3 (a). The impact force–displacement relation is as shown in Figure 3 (b) and it can be represented as [8].

$$F_c = k(u_1 - u_2 - g_p) + c(\dot{u}_1 - \dot{u}_2); u_1 - u_2 - g_p \geq 0$$

$$F_c = 0; u_1 - u_2 - g_p < 0$$
(1)

Where  $(\dot{u}_1 - \dot{u}_2)$  denotes the relative velocity of the structural elements undergoing collision and  $c$  is the impact

element's damping. By equating the energy losses during impact, the damping coefficient  $c$  can be related to the coefficient of restitution ( $e$ ) [8].

$$c = 2\zeta \sqrt{k \frac{m_1 m_2}{m_1 + m_2}}; \quad \zeta = \frac{\ln e}{\sqrt{\pi^2 + (\ln e)^2}} \quad (2)$$

Where  $m_1$  and  $m_2$  are the masses of the two colliding bodies and  $\zeta$  denotes the damping ratio related to the coefficient of restitution. The viscous impact damper of the Kelvin–Voigt model dissipates energy throughout the approach and restitution phases, but in actual fact, during the approach period large amount of energy dissipation takes place. On the other hand, at the restitution period small amount of energy is only dissipated. Due to the viscous damping term, Kelvin Voigt model displays an initial jump in the contact force because of the presence of viscous damping term. During the end of restitution phase, the damping force causes negative (tensile) contact force. This force draws the colliding bodies together. This is considered impractical for most of the structural materials.

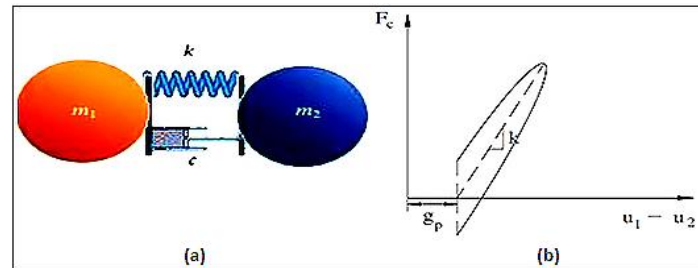


Figure 3: (a) Linear Viscoelastic model [15] (b) Force-displacement graph of linear spring model [8]

### 2.2.3 Modified linear viscoelastic model

The modified linear viscoelastic model was proposed by many authors in different ways to overcome the limitations of the linear viscoelastic model.

- i. Komodoros model [6]: On a small adjustment in linear viscoelastic model, the impractical existence of negative contact forces after the detachment of the colliding bodies is corrected in this model. By decreasing the impact force at the end of restitution period, when the contact is lost, the modified viscoelastic impact model restricts the tensile forces presuming small constant plastic deformations. However, the pounding force equation for modified linear viscoelastic model is same as that of equation (1).
- ii. Ye model [16][17]: A different modification to the Kelvin-Voigt model was proposed by Ye et.al. Ye model is capable of reflecting the physical nature of structural pounding better than Kelvin–Voigt model. The damping coefficient  $c$  and the damping constant  $\zeta$  are given by the following equations [16]:

$$c = \zeta (\dot{u}_1 - \dot{u}_2)$$

$$\zeta = \left(\frac{3}{2}\right) \frac{k(1-e)}{e(v_1 - v_2)}$$

Where  $(v_1 - v_2)$  is the relative impact velocity of the colliding bodies prior to the collision. The existence of negative tensile force immediately before detachment is not often avoided in this model [9]. The contact force-time relation and contact force penetration relation is as shown in Figure 4.

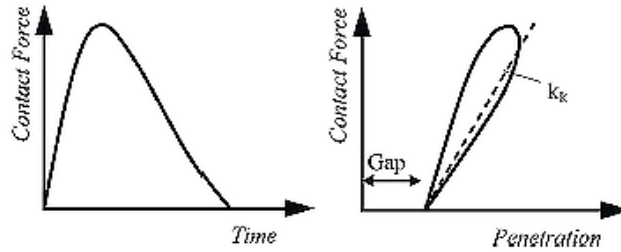


Figure 4: The modified linear viscoelastic model proposed by Ye et.al. [9]

- iii. Jankowski model [11]: Jankowski et.al. proposed another variation in linear viscoelastic model to remove the clinging tensile force that comes prior to the detachment of colliding structures. The damping term triggers only during the approaching period as shown in Figure 5. The pounding force during impact,  $F_c$ , for this model is given by the formula [11].

$$F_c = k(u_1 - u_2 - g_p) + c(\dot{u}_1 - \dot{u}_2) ; \dot{u}_1 - \dot{u}_2 > 0$$

$$F_c = k(u_1 - u_2 - g_p) ; \dot{u}_1 - \dot{u}_2 \leq 0$$

Since impact damping ratio and coefficient of restitution relation from equation (2) is invalid. The equation was reevaluated to satisfy the relation between the post-impact and the prior-impact relative velocities, as follows [11].

$$\zeta = \frac{(1 - e^2)}{e(e(\pi - 2) + 2)}$$

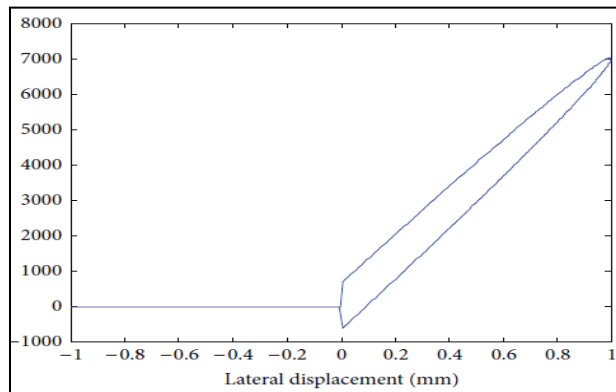


Figure 5: Force-displacement relationship for modified linear viscoelastic model proposed by Jankowski et.al. [11]

### 2.2.4 Hertz non-linear elastic model

The Hertz model contains a non-linear spring of stiffness ( $k_b$ ), as illustrated in Figure 6 (a). The impact force–displacement relation, as shown in Figure 6 (b) can be represented as [8].

$$F_c = k_b(u_1 - u_2 - g_p)^n ; u_1 - u_2 - g_p > 0$$

$$F_c = 0 ; u_1 - u_2 - g_p \leq 0$$

The contact force is directly proportional to the contact area between two colliding bodies. This leads to the development of a non-linear stiffness which can be represented by the non-linear coefficient ( $n$ ). The impact

stiffness parameter  $k_b$  depends on the material properties of the colliding structures and the contact surface geometry. The non-linear coefficient (n) is usually taken as 3/2. The drawback of the Hertz contact law model is that, this model is completely elastic and it does not consider the energy dissipation at the time of contact because of plastic deformations, friction, local cracking, etc [17].

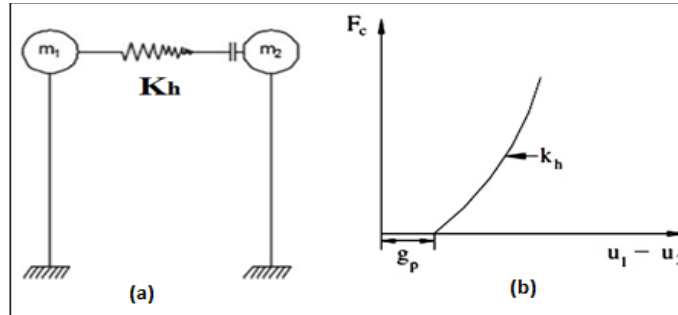


Figure 6: (a) Hertz non linear elastic model [7] (b) Force-displacement graph of Hertz non linear elastic model [8]

**2.2.5 Hertz model with non-linear damper (Hertz damp model)**

The drawbacks of Hertz non-linear elastic model was eliminated in Hertz damp model. In this model, a non-linear damper of damping coefficient  $c_b$  is used in conjunction with the non-linear spring of stiffness  $k_b$  as shown in Figure 7 (a). The impact force–displacement relation, as shown in Figure 7 (b) can be represented as [8]:

$$F_c = k_b(u_1 - u_2 - g_p)^n + c_b(\dot{u}_1 - \dot{u}_2) ; u_1 - u_2 - g_p > 0$$

$$F_c = 0 ; u_1 - u_2 - g_p \leq 0$$

The damping coefficient is taken as follows [8]:

$$c_b = \zeta(\dot{u}_1 - \dot{u}_2)^n$$

$$\zeta = \left(\frac{3}{4}\right) \frac{k_h(1 - e^2)}{e(v_1 - v_2)}$$

Where  $\zeta$  is the damping ratio and  $v_1 - v_2$  is the relative approaching velocity.  $n$  is taken as 3/2.

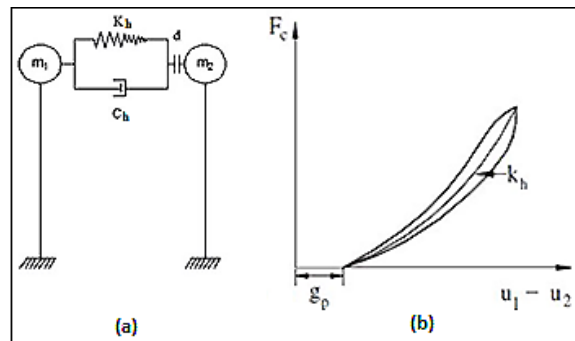


Figure 7: (a) Hertz damp model [7] (b) Force-displacement graph of Hertz damp model [8]

**2.2.6 Non-linear viscoelastic model**

The non-linear stiffness  $k_b$  is applied together with the non-linear damper having damping coefficient  $c_b$  [10]. During the approach period of collision it gets activated as shown in Figure 8 (a). During approach period in

this model, the process of energy dissipation is more accurately simulated. In the restitution period, the energy dissipation is omitted. The impact force–displacement relation, as shown in Figure 8 (b) can be represented as [10].

$$F_c = k_b(u_1 - u_2 - g_p)^n + c_b(\dot{u}_1 - \dot{u}_2) ; \dot{u}_1 - \dot{u}_2 > 0$$

$$F_c = k_b(u_1 - u_2 - g_p)^n ; \dot{u}_1 - \dot{u}_2 \leq 0$$

Impact element's damping coefficient is represented as follows [10].

$$c = 2\zeta \sqrt{k_h \sqrt{(u_1 - u_2 - g_p) \frac{m_1 m_2}{m_1 + m_2}}}$$

$$\zeta = \frac{9\sqrt{5}}{2} \frac{(1 - e^2)}{e(e(9\pi - 16) + 16)}$$

Where  $\zeta$  is the damping ratio and  $e$  is the coefficient of restitution.  $m_1$  and  $m_2$  are masses of the colliding bodies.

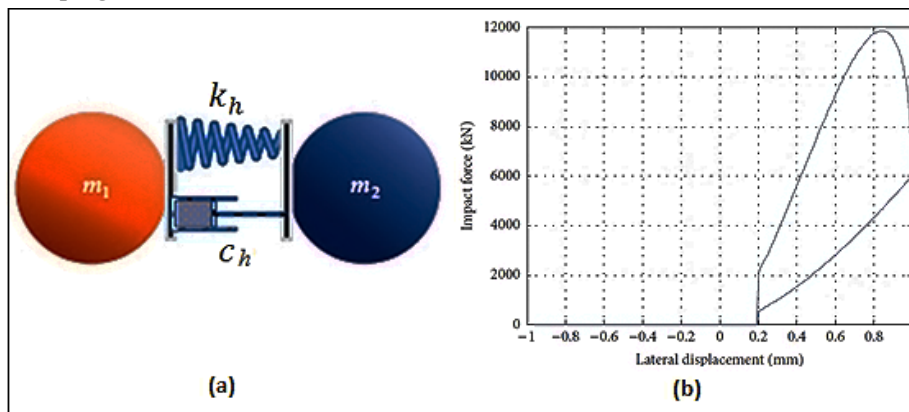


Figure 8: (a) Non linear viscoelastic model [15] (b) Force-displacement graph of non linear viscoelastic model [11]

### 2.2.7 Wang model

In case of low-speed pounding between viscoelastic material and steel, an advanced impact force model was introduced by Wang et.al. (2017) [14]. Unlike from the pounding between two hard materials, even if the impact took place at a low relative velocity a relaxation phenomenon called the elastic after-effect evidently takes place in pounding between a hard material, like steel and a soft material, like VE material. The Wang model consists of a non linear spring of stiffness  $k_b$  and a non linear viscous damper having damping ratio  $\zeta$  as shown in Figure 9. Huge energy loss occurs during the approach period of pounding. Therefore energy loss occurring at the restitution period is ignored in Wang model. At the approach period itself, the non-linear viscous damper is activated. At the final stage of pounding, the remaining surface deformation  $\delta_e$  happening on the surface of the viscoelastic materials layer is introduced in this model in order to consider the elastic after-effect phenomenon. The contact force equation proposed for low-speed pounding between viscoelastic materials and steel is given as follows [14].

$$F_c = k_b \delta^n + \zeta \delta^n \dot{\delta} ; \delta_{max} \geq \delta > 0, \dot{\delta} \geq 0$$

$$F_c = f_e \left( \frac{\delta - \delta_e}{\delta_{max} - \delta_e} \right)^n ; \delta_{max} > \delta \geq \delta_e, \dot{\delta} < 0$$

$$0 ; \delta_e > \delta > 0, \dot{\delta} < 0$$

Where  $\delta$  is the relative penetration between the colliding bodies;  $\dot{\delta}$  is the relative velocity of colliding bodies;  $\delta_e$  is the remaining surface deformation;  $\delta_{max}$  is the maximum relative penetration in the process of pounding and  $f_e$  is the maximum elastic force during pounding.

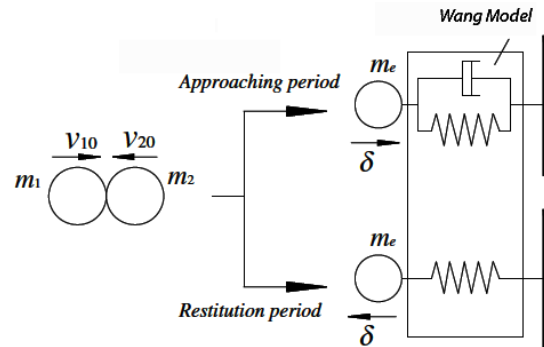


Figure 9: Equal model of the Wang model for pounding between 2 masses. [14]

### 3 Comparative study between Wang model and the existing viscoelastic force models

A comparative study between the Wang model and other existing viscoelastic models were carried out in order to display the performance of the Wang model [14]. The normalized error of all models was calculated. The lowest value for normalised error among all the force models was for Wang model, which is 26.32% as shown in figure 10.

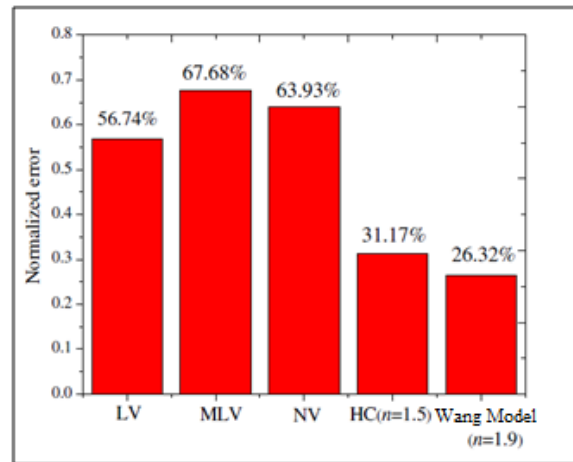


Figure 10: Normalized error of impact force models [14]

### 4 Conclusions

In this review paper, a comparative study of numerical models used for seismic induced pounding of building frames were carried out to select the best suited model. In addition to the common numerical models, Wang et.al. [14] introduced a model (Wang model) to analyse low speed pounding between viscoelastic material and steel is also put through as a case study in this review paper. The following conclusions were made from this study.

- The Hertz damp model is capable of modeling the energy loss. Also this model shows the least variation in system response to changes in coefficient of restitution.
- Linear viscoelastic model and non linear viscoelastic models have traction effect at the end of pounding.



- The linear viscoelastic, non linear viscoelastic and Hertz damp model is capable of modeling the pounding force - time curve during a single impact with least errors.
- For low speed pounding (at range from 0.025 to approximately 0.15m/s) for such cases, the best suited model is the Wang model.
- In general, Hertz damp model is considered to be the best model for simulating the pounding behaviour.
- The force-based pounding models are found to be more efficient to simulate pounding.

### How to Cite this Article:

Meghna, V., & Nisha, A. S. (2021). Comparison of Analytical Models for Seismic Induced Pounding in Buildings—A Review. *AIJR Proceedings*, 310-318.

### References

1. Anagnostopoulos (1988). *Pounding of buildings in series during earthquakes*. Earthquake Eng. and Structural Dynamics, 16, 443–456.
2. Chau K.T, Wei X.X, Guo X and Shen C.Y (2003). *Experimental and theoretical simulations of seismic poundings between two adjacent structures*. Earthquake Eng. and Structural Dynamics, 32, 537–554.
3. Bruce F Maison and Kazuhiko Kasai (1990). *Analysis for Type of Structural Pounding*, Journal of Structural Engineering. 116, 957-977
4. Chenna Rajaram and Ramancharla Pradeep Kumar (2012). *Pounding Between Adjacent Buildings: Comparison of Codal Provisions*. Indian Concrete Journal (ICJ), 86, 49- 59.
5. Chenna Rajaram and Ramancharla Pradeep Kumar (2014). *Three dimensional analysis of pounding between adjacent buildings*. Journal of Structural Engineering, 41, 1-11.
6. Komodromos P. et. al. (2007). *Response of seismically isolated buildings considering poundings*. Earthq.Eng. Struct.Dyn. 36, 1605–1622.
7. Mahmoud Miari, Kok Keong Choong and Robert Jankowski (2018). *Seismic Pounding between adjacent buildings: Identification of parameters, soil interaction issues and mitigation measures*. Soil Dynamics and Earthquake Engineering, 121, 135-150.
8. Muthukumar S. et.al. (2006). *A Hertz contact model with nonlinear damping for pounding simulation*, Earthquake Engineering and Structural Dynamics, 35, 811–828.
9. Polycarpou P.C and Komodromos P (2012). *An efficient methodology for earthquake induced 3D pounding of buildings*. Earthquake Engineering & Structural Dynamics, 39, 933–940.
10. Robert Jankowski (2004). *Non-Linear Modelling of Earthquake Induced Pounding Between Buildings*. Mechanics of 21<sup>st</sup> Century-ICTAM04 Proceedings.
11. Robert Jankowski and Sayed Mahmoud (2008). *Earthquake Induced Pounding Between Equal Height Buildings with Substantially Different Dynamic Properties*. Engineering Structures, 30, 2818-2829.
12. Robert Jankowski and Sayed Mahmoud (2015). *Earthquake induced structural pounding*. The GeoPlanet: Earth and Planetary Sciences, Springer.
13. Susender Muthukumar et.al. (2004) Evaluation of impact models for pounding, Proceedings on the 13<sup>th</sup> World Conference on Earthquake Engineering.
14. Wenxi Wang et.al. (2017). *Advanced Impact Force Model for Low-Speed Pounding between Viscoelastic Materials and Steel*, Journal of engineering mechanics, 134, 145-153.
15. Xu et.al. (2016). *An Updated Analytical Structural Pounding Force Model Based on Viscoelasticity of Materials*. Journal of shock and vibrations, 123,146-158.
16. Ye K., Li L., and Zhu H. (2009a). *A modified Kelvin impact model for pounding simulation of base-isolated building with adjacent structures*, Earthq. Eng. Eng. Vib. 8, 433–446.
17. Ye K., Li L., and Zhu H. (2009b). *A note on the Hertz contact model with non linear damping for pounding simulation*. Earthq. Eng. Struct.Dyn, 38, 1135–1142.