Lie group analysis for Elastico-viscous Maxwell nanofluid flow in a channel of stretching surface with convective boundary condition

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ABSTRACT

The current study deals with the analysis of heat transfer of the unsteady Maxwell nanofluid flow in a channel of a porous extensile surface subject to the convective boundary condition. The Lie group analysis is performed for the transformation of the current model in a system of nonlinear ordinary differential equations that are numerically decoded with the help of MATLAB integrated function bvp4c. The effects of various flow control parameters are investigated for the momentum, temperature and diffusion profiles, as well as for the wall shearing stress and heat transfer presuming two cases, prescribed surface temperature (PST) and prescribed convective boundary (PCB). Finally, the results are described from the material point of view. In general, the PST and PCB boundary conditions are highly functional in various industrial, biological and engineering applications. In addition, a significant result of the current analysis is that the viscosity of the nanofluid increases with the gradual increase in the Deborah number, which increases the resistance to flow and there is a transverse flow in the channel near the stretching surface.

Keyword: Nanofluid, Deborah number, Concentration, Convective surface, Suction, Unsteady flow.

1 Introduction

Nanofluids are a newly invented heat transfer fluids having higher thermal conductivity at low particle diffusions compared to conventional fluids. They are made by steadily suspending and consistently dispersing a few of ultrafine, nonmetallic or nano-scale metallic particles in ordinary heat transfer fluids. This idea of nanofluid was established by Choi [1]. Recent investigators have discovered that replacing conservative refrigerants with nanofluids can be beneficial in processes such as improving heat transfer efficiency in nuclear and engineering environments, household refrigerators / freezers; and cooling of automobile engines [2-7]. A mathematical model is developed by Buongiorno [8] to study the transfer of heat in nanofluids by convection taking into account two essential effects, namely Brownian diffusion and thermophoresis diffusion.

In addition, magneto-hydrodynamics (MHD) is the relationship between electromagnetic fields and conductive fluids. Numerous applications of nanofluids are found in the areas of manufacturing, engineering and science mainly in the design of heat exchange devices, accelerators and MHD generators, etc. [9-12].

A captivating and highly unresolved tribological matter refers to the influence of elastico-viscosity on lubrication phenomena in thin film flows. The practice of adding polymers to mineral oils, known as multi-grade oils, has been recognized since the mid-1990s [13-15]. These additions force the resulting lubricants to become non-Newtonian and viscoelastic by exerting a viscosity dependent on the shear rate [16,17]. The

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classic Newtonian fluid model that contains the Navier-Stokes equations cannot demonstrate the highly non-linear connection between shear stress and strain rate of non-Newtonian fluids [18-20]. Maxwell fluid is a simple class of rate-type viscoelastic material that has the features of the fluid relaxation phase, namely the viscosity - modulus of elasticity ratio. It eliminates the complex effects of shear-related viscosity and thus makes it possible to demonstrate the influence of fluid elasticity on the characteristics of its boundary layer [19]. Harris [13] developed for the first time the constitutive equation of the upper convected Maxwell Fluid (UCM) in order to model the lubricant behavior of non-Newtonian fluid. Due to the proliferation of practical applications in industrial and manufacturing procedures, such as bioengineering and plastic manufacturing, paper production, food processing and aerodynamic extrusion of plastic films, researchers have increased attention to the investigation of boundary layer flows of non-Newtonian fluids. [21,22].

Norwegian mathematician Sophus Lie developed a classic scheme called the Lie group transformation to discover invariant and similarity of solutions [23-27]. Lie group analysis provides an appropriate method to address nonlinear equations. The Lie group transformation proposes a precise mathematical formulation of perceptive thoughts of symmetry and offers beneficial techniques for the analytical resolution of nonlinear differential equations. Lie group analysis is an emergent field of mathematics with many applications. Researchers are currently analyzing the transformations of Lie groups for Newtonian and non-Newtonian fluid flows [26, 28].

Though the Maxwell nanofluid model can explain the time dependent stress relaxation of viscoelastic fluid hosting solid nanoparticles, the non-Newtonian fluid with enhanced heat transfer has a great opportunity to contribute to industrial and hemodynamic purposes. By observing previous research, the prime interest is to analyze the time dependent two-dimensional laminar flow of elastico-viscous nanofluid in a convective boundary porous stretched surface channel incorporating the Maxwell's rheological fluid using the Lie group transformations.

2 Mathematical model

In the extant model, an unsteady movement of an elastico-viscous Maxwell nanofluid with the effect of magnetic field passing through a channel of stretching porous surface with convective boundary conditions is considered. To explain the somatic model, the Cartesian co-ordinate system is introduced where x-axis is taken along the channel surfaces and y-axis is vertical to the parallel channel surfaces, shown in Fig. 1.

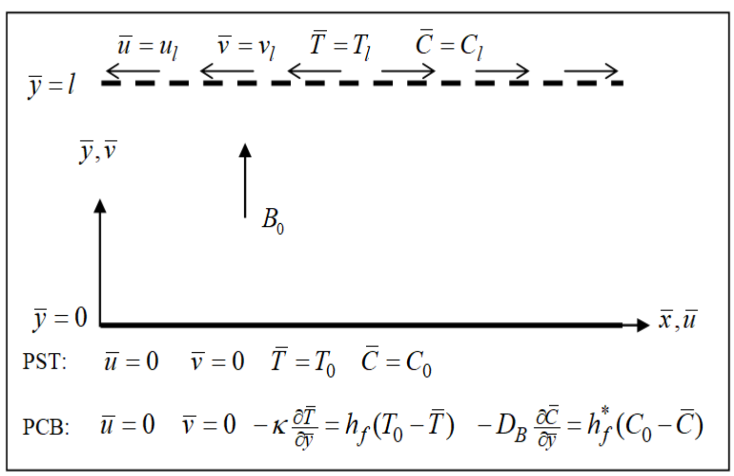

Fig. 1. Physical model.

When elastic stress is functional to a non-Newtonian fluid, the resultant strain is dependent on time, which is illustrated by the relaxation time factor. The constitutive equation for a Maxwell fluid following Fetecau and Fetecau [19] is

$$\tau = -p\mathbf{I} + \mathbf{S} \tag{1}$$

where τ is the Cauchy stress tensor and the extra stress tensor $\mathbf{S}$ satisfies

$$\mathbf{S} + \lambda_S\left(\frac{d\mathbf{S}}{dt} - \mathbf{LS} - \mathbf{SL}^{tr}\right) = \mu_0\left(\nabla\mathbf{V} + (\nabla\mathbf{V})^{tr}\right) \tag{2}$$

in which μ_0 is the viscosity, $\lambda_S > 0$ is the fluid relaxation time. Here, $\mathbf{V}$ means the velocity of the fluid. Then the equations of flow are given by

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \tag{3}$$

$$\begin{aligned}\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} + \lambda_S\left(\frac{\partial^2\bar{u}}{\partial\bar{t}^2} + 2\bar{u}\frac{\partial^2\bar{u}}{\partial\bar{t}\,\partial\bar{x}} + 2\bar{v}\frac{\partial^2\bar{u}}{\partial\bar{t}\,\partial\bar{y}} + \bar{u}^2\frac{\partial^2\bar{u}}{\partial\bar{x}^2} + \bar{v}^2\frac{\partial^2\bar{u}}{\partial\bar{y}^2} + 2\overline{uv}\frac{\partial^2\bar{u}}{\partial\bar{x}\partial\bar{y}}\right)\\ = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{x}} + \nu\frac{\partial^2\bar{u}}{\partial\bar{y}^2} - \frac{\sigma B_0{}^2}{\rho}\left(\bar{u} + \lambda_S\left(\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right)\right)\end{aligned} \tag{4}$$

Again, considering the Buongiorno model integrating the combined effects of thermophoresis and Brownian diffusions [8, 29], the equations of energy and diffusion are

$$\frac{\partial\bar{T}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{T}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{T}}{\partial\bar{y}} = \frac{\kappa}{\rho c_p}\frac{\partial^2\bar{T}}{\partial\bar{y}^2} + \tau\left(D_B\frac{\partial\bar{T}}{\partial\bar{y}}\frac{\partial\bar{C}}{\partial\bar{y}} + \frac{D_T}{T_a}\left(\frac{\partial\bar{T}}{\partial\bar{y}}\right)^2\right) + \frac{\mu}{\rho c_p}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^2 \tag{5}$$

$$\frac{\partial\bar{C}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{C}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{C}}{\partial\bar{y}} = D_B\frac{\partial^2\bar{C}}{\partial\bar{y}^2} + \frac{D_T}{T_a}\frac{\partial^2\bar{T}}{\partial\bar{y}^2} \tag{6}$$

The solutions of the model depend on the nature of the prescribed thermal boundary condition [21]. The corresponding boundary conditions are:

Case 1: Prescribed surface temperature (PST)

$$\left.\begin{aligned}&\bar{u} = u_l,\ \bar{v} = v_l,\ \bar{T} = T_l,\ \bar{C} = C_l \quad \text{at } \bar{y} = l\\ &\bar{u} = 0,\ \ \bar{v} = 0,\ \ \bar{T} = T_0,\ \bar{C} = C_0 \quad \text{at } \bar{y} = 0\end{aligned}\right\} \tag{7}$$

Case 2: Prescribed convective boundary (PCB) case

$$\left.\begin{aligned}&\bar{u} = \gamma_h u_w, \bar{v} = v_h, \quad \bar{T} = T_l, \qquad\qquad \bar{C} = C_l \qquad \text{at } \bar{y} = l\\ &\bar{u} = 0, \quad \bar{v} = 0, -\kappa\tfrac{\partial\bar{T}}{\partial\bar{y}} = h_f(T_l - \bar{T}),\ -D_B\tfrac{\partial\bar{C}}{\partial\bar{y}} = h_f^*(C_l - \bar{C}) \quad \text{at } \bar{y} = 0\end{aligned}\right\} \tag{8}$$

where λ_S is the stress relaxation time, p is the nanofluid pressure, D_B is the Brownian diffusion, D_T is the thermophoresis diffusion, B_0 is the magnetic field strength applied in the y-axis, u_l is the stretching velocity, v_l is the suction velocity, $T_l = T_0 + (T_h - T_0)\frac{x}{(1-t)^2}$ and $C_l = C_0 + (C_h - C_0)\frac{x}{(1-t)^2}$ are the temperature and the

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concentration respectively at the upper plate supposed to vary along the surface and in time, T_0 and C_0 are the temperature and concentration at bottom the plate and h_f , h_f^* are convective heat transfer coefficient and convective mass transfer coefficient respectively. Here $\mu, \nu, \rho, \kappa, \alpha, C_p$ and σ are the dynamic and kinematic viscosity, density, thermal conductivity, thermal diffusivity, specific heat and electrical conductivity respectively.

The following dimensionless variables are being introduced

$$t = \frac{\bar{t}u_w}{h} \quad x = \frac{\bar{x}}{h} \quad y = \frac{\bar{y}}{h} \quad u = \frac{\bar{u}}{u_w} \quad v = \frac{\bar{v}}{u_w} \quad p = \frac{\bar{p}}{p_0} \quad T = \frac{\bar{T} - T_0}{T_h - T_0} \quad C = \frac{\bar{C} - C_0}{C_h - C_0} \tag{9}$$

Then introducing the above variables in equations (4)–(6), the dimensionless PDE model is given by the following equations

$$\begin{aligned} &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{u_w \lambda_S}{h}\left(\frac{\partial^2 u}{\partial t^2} + 2u\frac{\partial^2 u}{\partial t \partial x} + 2v\frac{\partial^2 u}{\partial t \partial y} + u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) \\ &\qquad = -\frac{p_0}{u_w{}^2 \rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{\mathrm{Re}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0{}^2 h}{\rho u_w}\left(u + \frac{u_w \lambda_S}{h}\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right)\right) \end{aligned} \tag{10}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}}\frac{\partial^2 T}{\partial y^2} + \frac{1}{\mathrm{Re}}\left(Nb_x\frac{\partial T}{\partial y}\frac{\partial C}{\partial y} + Nt_x{\frac{\partial T}{\partial y}}^2\right) + \frac{Ec_x}{\mathrm{Re}}{\frac{\partial u}{\partial y}}^2 \tag{11}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{1}{\mathrm{Re}\,Le}\frac{\partial^2 C}{\partial y^2} + \frac{Nt_x}{\mathrm{Re}\,Le\,Nb_x}\frac{\partial^2 T}{\partial y^2} \tag{12}$$

Here the dimensionless parameters, $\mathrm{Re} = \frac{u_w h}{\nu}$ is the Reynolds number, $\mathrm{Pr} = \frac{\nu \rho c_p}{\kappa}$ is the Prandtl number; $Nb_x = \frac{\tau D_B (C_h - C_0)}{\nu}$ is the Brownian motion parameter; $Ec_x = \frac{u_w{}^2}{c_p (T_h - T_0)}$ is the Eckert number; $Nt_x = \frac{\tau D_T (T_h - T_0)}{\nu T_a}$ is the thermophoresis parameter; $Le = \frac{\nu}{D_B}$ is the Lewis number.

Set $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ in the above equations to get

$$\begin{aligned} &\frac{\partial^2 \psi}{\partial t \partial y} + \frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} + \\ &\frac{u_w \lambda_S}{h}\left(\frac{\partial^3 \psi}{\partial t^2 \partial y} + 2\frac{\partial \psi}{\partial y}\frac{\partial^3 \psi}{\partial t \partial x \partial y} - 2\frac{\partial \psi}{\partial x}\frac{\partial^3 \psi}{\partial t \partial y^2} + \left(\frac{\partial \psi}{\partial y}\right)^2\frac{\partial^3 \psi}{\partial x^2 \partial y} + \left(\frac{\partial \psi}{\partial x}\right)^2\frac{\partial^3 \psi}{\partial y^3} - 2\frac{\partial \psi}{\partial y}\frac{\partial \psi}{\partial x}\frac{\partial^3 \psi}{\partial x \partial y^2}\right) \\ &\qquad = -\frac{p_0}{u_w{}^2 \rho}\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma B_0{}^2 h}{\rho u_w}\left(\frac{\partial \psi}{\partial y} + \frac{u_w \lambda_S}{h}\left(\frac{\partial^2 \psi}{\partial t \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}\right)\right) \end{aligned} \tag{13}$$

$$\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2} + \frac{1}{\mathrm{Re}}\left(Nb_x\frac{\partial T}{\partial y}\frac{\partial C}{\partial y} + Nt_x\left(\frac{\partial T}{\partial y}\right)^2\right) + \frac{Ec_x}{\mathrm{Re}}\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2 \tag{14}$$

$$\frac{\partial C}{\partial \bar{t}}+\frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y}=\frac{1}{\operatorname{Re} Le}\frac{\partial^2 C}{\partial y^2}+\frac{Nt_x}{\operatorname{Re} Le\, Nb_x}\frac{\partial^2 T}{\partial y^2} \tag{15}$$

Dimensionless prescribed surface temperature (PST) boundary conditions are

$$\left.\begin{array}{llll} \frac{\partial \psi}{\partial y}=\frac{u_l}{u_w}=\gamma_l, & \frac{\partial \psi}{\partial x}=-\frac{v_l}{u_w}, & T=\frac{x}{(1-t)^2},\ C=\frac{x}{(1-t)^2} & \text{at } y=\frac{l}{h} \\ \frac{\partial \psi}{\partial y}=0, & \frac{\partial \psi}{\partial x}=0, & T=0,\ C=0 & \text{at } y=0 \end{array}\right\} \tag{16}$$

3 Method of transformation

The importance of similarity solutions in many areas of science and engineering is limitless. It is compulsory to seek a universal symmetrical approach applicable to certain mathematical models. The Lie group analysis transformation procedure is a solid technique for the theory of continuous symmetry of numerical structures which is immensely functional for various fields of modern mathematical physics. This analysis should provide a new methodology for studying the continuous symmetries of the model equations governing heat transfer fluxes in Maxwell nanofluids. In progress, this theory reduces the number of independent parameters of the governing partial differential equations considered for the physical model and maintains the invariant structure of the model with the corresponding initial conditions and limits. Due to the unsteady flow ($0 < t < 1$) of fluid in a channel, the following two parameters of groups of linear transformations are to be considered for the analysis of Lie groups (Uddin *et al.* [30]):

$$\begin{aligned}\Gamma:\ & \hat{t}=(1-t)e^{\alpha_1},\ \hat{x}=xe^{\beta_1},\ \hat{\psi}=\psi e^{\alpha_2}e^{\beta_2},\ \hat{y}=ye^{\alpha_3}e^{\beta_3},\ \hat{T}=Te^{\alpha_4}e^{\beta_4},\ \hat{C}=Ce^{\alpha_5}e^{\beta_5}\\ & \hat{p}=pe^{\alpha_6}e^{\beta_6},\ \hat{\lambda}_S=\lambda_S e^{\alpha_7}e^{\beta_7},\ \hat{B}_0=B_0e^{\alpha_8}e^{\beta_8},\ E\hat{c}=Ec_x e^{\alpha_9}e^{\beta_9},\ \hat{\gamma}_l=\gamma_l e^{\alpha_{10}}e^{\beta_{10}}\\ & \hat{v}_l=v_l e^{\alpha_{11}}e^{\beta_{11}},\ \hat{l}=le^{\alpha_{12}}e^{\beta_{12}},\ \hat{N}b=Nb_x e^{\alpha_{13}}e^{\beta_{13}},\ \hat{N}t=Nt_x e^{\alpha_{14}}e^{\beta_{14}}\end{aligned} \tag{17}$$

where α_i, β_i (i = 1, 2, . . . , 14) are constants. We seek the values of α_i, β_i so that the Eqs (13)–(15) are invariant under the transformations connected by the following relations

$$\begin{array}{llllll} \alpha_2=-\frac{1}{2}\alpha_1 & \alpha_3=\frac{1}{2}\alpha_1 & \alpha_4=\alpha_5=-2\alpha_1 & \alpha_6=-2\alpha_1 & \alpha_7=\alpha_1 & \alpha_8=-\frac{1}{2}\alpha_1\\ \alpha_9=0 & \alpha_{10}=-\alpha_1 & \alpha_{11}=-\frac{1}{2}\alpha_1 & \alpha_{12}=\frac{1}{2}\alpha_1 & \alpha_{13}=2\alpha_1 & \alpha_{14}=2\alpha_1\\ \beta_2=\beta_1 & \beta_3=0 & \beta_4=\beta_5=\beta_1 & \beta_6=2\beta_1 & \beta_7=\beta_8=0 & \\ \beta_9=-\beta_1 & \beta_{10}=\beta_1 & \beta_{11}=\beta_{12}=0 & \beta_{13}=\beta_{14}=\beta_1 & & \end{array} \tag{18}$$

With these relationships of α_i, β_i, Eq (17) turns into

$$\begin{aligned}\Gamma:\ & \hat{t}=(1-t)e^{\alpha_1},\ \ \hat{x}=xe^{\beta_1},\ \ \hat{\psi}=\psi e^{-\frac{1}{2}\alpha_1}e^{\beta_1},\ \ \hat{y}=ye^{\frac{1}{2}\alpha_1},\ \hat{T}=Te^{-2\alpha_1}e^{\beta_1}\\ & \hat{C}=Ce^{-2\alpha_1}e^{\beta_1},\ \ \hat{p}=e^{-2\alpha_1}e^{2\beta_1},\ \hat{\lambda}_S=\lambda_S e^{\alpha_1},\ \hat{B}_0=B_0e^{-\frac{1}{2}\alpha_1},\ E\hat{c}=Ec_x e^{-\beta_1}\\ & \hat{\gamma}_l=\gamma_l e^{-\alpha_1}e^{\beta_1},\ \ \hat{v}_l=v_l e^{-\frac{1}{2}\alpha_1},\ \hat{l}=le^{\frac{1}{2}\alpha_1},\ \hat{N}b=Nbe^{2\alpha_1}e^{-\beta_1},\ \ \hat{N}t=Nt\,e^{2\alpha_1}e^{-\beta_1}\end{aligned} \tag{19}$$

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From the absolute invariant relations, the similarity parameters are described as

$$\begin{gathered}\eta=(1-t)^{-\frac{1}{2}}y,\;\; \psi=(1-t)^{-\frac{1}{2}}xf(\eta),\;\; T=(1-t)^{-2}x\theta(\eta),\;\; C=(1-t)^{-2}xF(\eta)\\ p=(1-t)^{-2}x^2p_\eta,\; \lambda_S=(1-t)\lambda,\;\; B_0=(1-t)^{-\frac{1}{2}}B,\;\; Ec_x=x^{-1}Ec,\;\; \gamma_l=(1-t)^{-1}x\gamma\\ v_l=(1-t)^{-\frac{1}{2}}v_w,\;\; l=\sqrt{1-t}\,h,\;\; Nb_x=(1-t)^2x^{-1}Nb,\;\; Nt_x=(1-t)^2x^{-1}Nt\end{gathered}\tag{20}$$

Using Eq (20) into Eqs (13)–(15), we obtain the following similarity equations,

$$\begin{gathered}\left(\frac{1}{\mathrm{Re}}-\frac{\beta_S}{2}\left(\eta^2-4\eta f+4f^2\right)\right)f^{iv}+\frac{1}{2}\left(\eta f'''+3f''+2f'f''-2ff'''\right)\\ -\frac{\beta_S}{2}\left(9\eta f'''+4\eta f''^2-16f\,f'''+8f'f''-8f\,f''^2-8f'^2f''+15f''\right)\\ -M\left(f''+\beta_S\left(\eta f'''+3f''-2ff'''-2ff''\right)\right)=0\end{gathered}\tag{21}$$

$$\theta''-\mathrm{Pr}\left(\mathrm{Re}\left(\frac{\eta}{2}\theta'+2\theta+f'\theta-f\theta'\right)+Nb\,\theta'F'+Nt\theta'^2\right)+\mathrm{Pr}\,Ecf''^2=0\tag{22}$$

$$F''-\mathrm{Re}\,Le\left(\frac{\eta}{2}F'+2F+f'F-f\,F'\right)+\frac{Nt}{Nb}\theta''=0\tag{23}$$

The transformed PST boundary conditions are

$$\left.\begin{array}{ll}f'=\gamma,\quad f=fw,\quad \theta=1,\quad F=1 & \text{at}\ \ \eta=1\\ f'=0,\quad f=0,\quad \theta=0,\quad F=0 & \text{at}\ \ \eta=0\end{array}\right\}\tag{24}$$

Following the similar procedure the PCB boundary conditions take the form

$$\left.\begin{array}{lll}f'=\gamma,\ f=f_w,\ \theta=1, & F=1 & \text{at}\ \eta=1\\ f'=0,\ f=0,\ \ \theta'=-Bi(1-\theta), & F'=-Bi^*(1-F) & \text{at}\ \eta=0\end{array}\right\}\tag{25}$$

$\beta_S=\frac{u_w\lambda_S}{2h}$ is the Maxwell parameter; $M=\frac{\sigma B_0{}^2h}{\rho u_w}$ is the Magnetic field parameter; γ is the stretching parameter; $f_w=-\frac{v_w}{u_w}$ is the suction parameter; $Bi=\frac{h_fh}{\kappa}$ and $B^*i=\frac{h^*{}_fh}{\kappa}$ are the Biot numbers for convective heat and mass transfer respectively.

Finally, the coefficient of skin friction estimates the friction force applied to the surface and the Nusselt number characterizes the heat flux from a heated surface to a fluid. The skin friction coefficient C_f and the local Nusselt number Nu are defined respectively as

$$C_f\propto f''(1)\qquad\text{and}\qquad Nu\propto-\theta'(1)\tag{26}$$

4 Numerical methods

Equations (21)–(23) combined with the boundary conditions (24) and (25) respectively are solved numerically using collocation method. The analysis is made for various parameters such as Maxwell parameter β, stretching parameter γ, suction parameter *fw*, magnetic field parameter *M*, Prandtl number Pr, Eckert number *Ec*, thermophoresis parameter *Nt*, Brownian motion parameter *Nb*, Lewis number *Le*. The mesh size is used as $\eta=0.01$ and the tolerance factor is set to 10^{-6}. On the basis of the present model, we are considering $[0,1]$ as the domain of the channel problem.

5 Results and Discussion

The renovation of the model equations trims down the mathematical work extensively. Graphical representations of consequences are very constructive to discuss the physical features offered by the solutions.

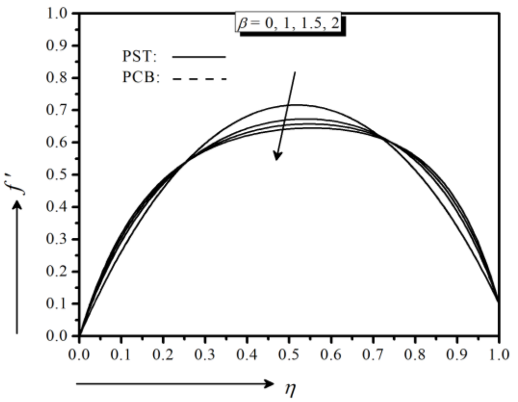

Fig. 2. Velocity distribution for various values of Deborah number β .

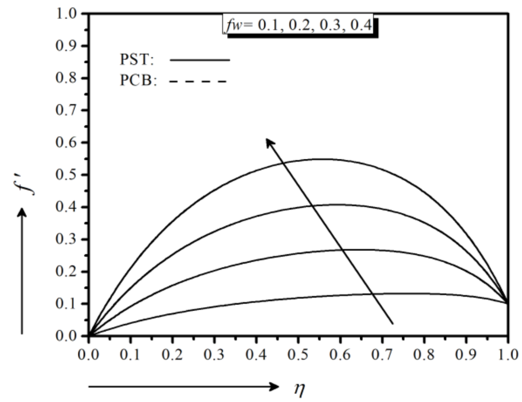

Fig. 5. Velocity distribution for various values of suction parameter *fw*.

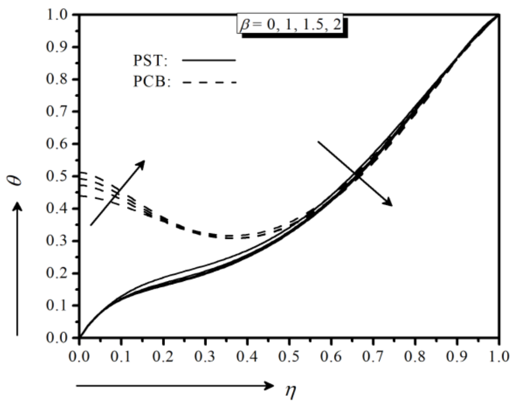

Fig. 3. Temperature distribution for various values of Deborah number β .

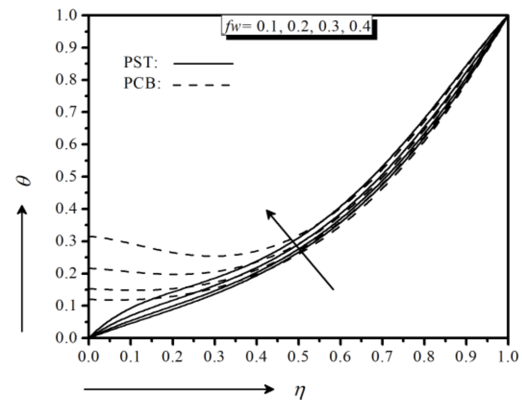

Fig. 6. Temperature distribution for various values of suction parameter *fw*.

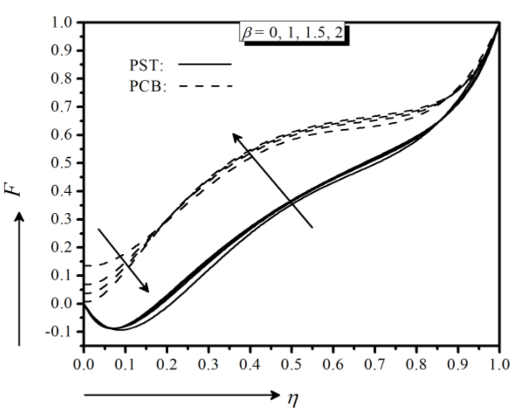

Fig. 4. Diffusion distribution for various values of Deborah number β .

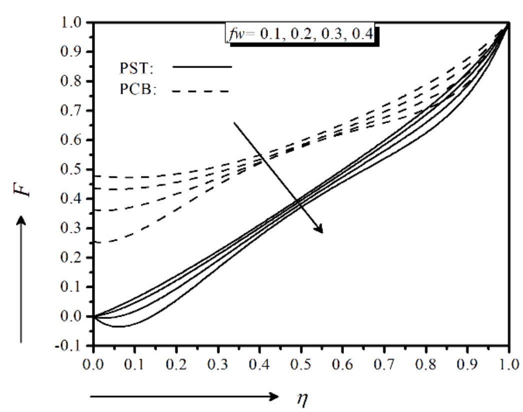

Fig. 7. Diffusion distribution for various values of suction parameter *fw*

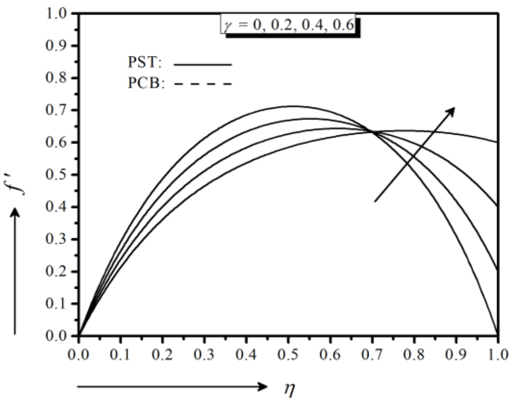

Fig. 8. Velocity distribution for various values of stretching parameter γ .

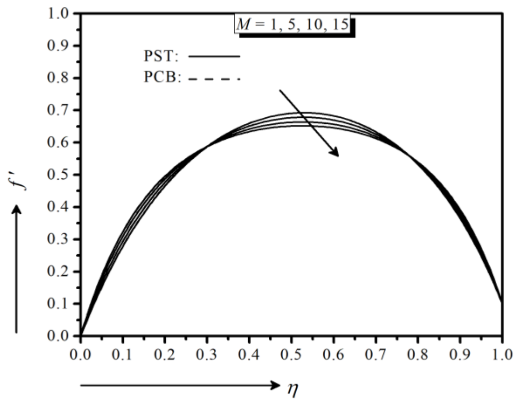

Fig. 11. Velocity distribution for various values of magnetic parameter *M.*

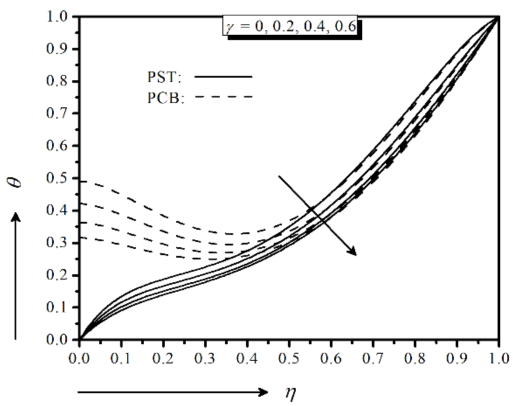

Fig. 9. Temperature distribution for various values of stretching parameter γ .

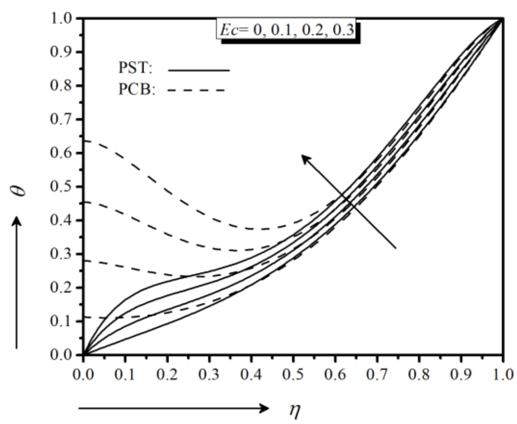

Fig. 12. Temperature distribution for various values of Eckert number *Ec.*

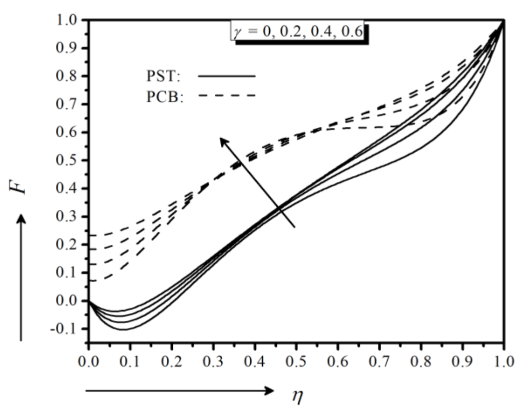

Fig. 10. Diffusion distribution for various values of stretching parameter γ .

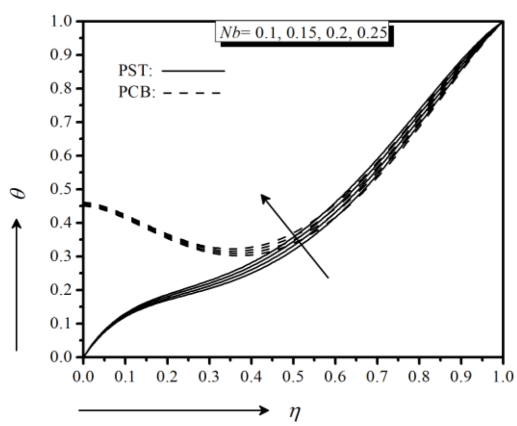

Fig. 13. Temperature distribution for various values of Brownian parameter *Nb.*

The time during which the non-Newtonian fluid gains stability after the application of an elastic stress is the relaxation time. This time factor is higher for highly viscous fluids. The stress relaxation factor β, named as the Deborah number, is a dimensionless variable that treats the fluid relaxation time to its characteristic time scale. Here, $\beta = 0$ gives the result for the viscous incompressible Newtonian fluid. Liquids with a small number of Deborah have a liquid-like behavior, while a large number of Deborah communicate with solids-like substances that can better conduct and preserve heat. Therefore, it is found that progressively increasing the Deborah number can enlarge the viscosity of the fluid, which increases the resistance to flow and, consequently, the thickness of the hydrodynamic boundary layer decreases for the Maxwell fluid, as revealed in Fig 2 for PST and PCB cases. The Deborah number decreases the thermal boundary layer thickness, but gives a slight upward effect for the PCB near the bottom plate, as illustrated in Fig 3. Figure 4 describes the rising effect of Deborah number β on the diffusion profile.

Figures 5–7 represent the effect of suction parameter *fw* on velocity, thermal energy and concentration distributions, respectively. It is apparent that velocity and energy distributions are increasing due to higher suction for both the PST and PCB boundary surface. Actually, the forced suction brings a force to the fluid in the channel which in turn improves the movement and energy within the fluid.

From Fig 8, it is observed that the stretching velocity parameter γ enhances strength to the fluid velocity to increase the momentum distribution with the increase of stretching effect near the upper surface. Figures 9–10 demonstrates the effect of stretching parameter on energy and diffusion distributions respectively.

The effects of magnetic field on momentum for both cases are demonstrated in Fig. 11. It is established that an enhancing magnetic parameter *M* trims down the velocity of fluids. This is the consequence of the effect of the magnetic field applied on an electrically conductive fluid, which creates a drag force called Lorentz force against the flow direction along the surface to slow down velocity. As a result, the magnetic field is liable for retarding the fluid flow.

Figure 12 illustrates that the temperature profiles increases with the increasing value of Eckert number *Ec* indicating that the dissipative effect of the fluid enhances the energy distribution. The Brownian diffusion develops the thermal transport in the fluid, as noticed in Fig. 13. The Eckert number *Ec* and Brownian motion parameter *Nb* increase heat transport in the fluid, which is an important mechanism for the enhancement of heat transfer in nanofluids.

Additionally, the effect of stress relaxation parameter Deborah number on the skin fraction coefficient (f'') and the local Nusselt number ($-\theta'$), from Eq (26), are arranged in the Table 1 considering Re = 1, $M = 1$, Pr = 6.838, $Ec = 0.2$, $Le = 5$, $Nb = 0.16$, $Nt = 0.4$, $\gamma = 0.1$, $fw = 0.5$, $Bi = 0.1$, and $Bi^* = 0.3$.

6 Conclusion

The major consequences drawn from the study of the present model can be summarized as follows:

- The temperature distribution of nanofluid is prominent in the state of the PCB.
- The viscous dissipation and Brownian diffusion develop the thermal boundary layer thickness.
- The Deborah number and magnetic field parameter are liable for reducing the hydrodynamic boundary layer thickness.
- The effects of stretching and suction parameters on the fluid velocity are significant.

In conclusion of the present study, it can be affirmed that this model presenting phenomena of velocity control and improvement of the transfer of heat in the liquid can be an excellent opportunity to develop the performances of cooling of the mechanical systems with less friction.

Table 1: Skin Friction f'' and heat transfer ($-\theta'$) with Parameters Variations for different values of Deborah number β .

Deborah number β	PST condition				PCB condition			
	Upper plate, $\eta = 1$		Lower plate, $\eta = 0$		Upper plate, $\eta = 1$		Lower plate, $\eta = 0$	
	f''	$-\theta'$	f''	$-\theta'$	f''	$-\theta'$	f''	$-\theta'$
0	2.96864	-1.92124	-2.72599	-1.13957	2.96864	0.04842	-2.72599	-1.19157
1	3.70291	-2.01223	-3.7475	-0.9088	3.70291	0.04553	-3.7475	-0.96326
1.5	4.02107	-2.07573	-4.19155	-0.80379	4.02107	0.04384	-4.19155	-0.85972
2	4.31336	-2.14192	-4.60863	-0.70325	4.31336	0.04214	-4.60863	-0.76064

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